Shewhart methodology for modelling financial series

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Abstract

Quality management techniques are widely used in industrial applications for monitoring observable process variation. Among them, the scientific notion of Shewhart principles is vital for understating variations in any type of process or service. This study extensively investigates and demonstrates Shewhart methodology for financial data.

Extremely heavy tails noted in the empirical distribution of stock returns led to the development of new parametric probability distributions for pricing assets and forecasting market risk. Standard asset pricing models have also extended to account the first four (excess) moments in return distributions. These approaches remain complex, but yet they are inadequate for capturing extreme volatility caused by infrequent market events.

It is well known that the security markets are always subjected to a certain amount of variability caused by noise-traders and other frictional price changes. Unforeseen events which are happening in the world may lead to huge market losses. This research shows that Shewhart methodology for partitioning data into common and special cause variations adds value to modelling stock returns.

Applicability of the proposed method is discussed using several scenarios occurring in an industrial process and a financial market. A set of new propositions based on Shewhart methodology is formed for finer description of the statistical properties in stock returns. Research issues which are related to the first four moments, co-moments and autocorrelation in stock returns are identified. New statistical tools such as difference control charts, odd-even analysis and estimates for co-moments are proposed to investigate the new propositions and research issues. Finally, several risk measures are proposed, and considered with respect to investor's preferences.

The research issues are investigated using partitioned data from S&P 500 stocks and the findings show that in most of the scenarios, contradictory conclusions were made as a result of special cause variations. A modelling approach based on common and special cause variations is therefore expected to lead appropriate asset pricing and portfolio management. New statistical tools proposed in this study can be used to other time series data; a new R-package called QCCTS (Quality Control Charts for Time Series) is developed for this purpose.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

Distributional properties, extreme fluctuations, linear and non-linear dependence in stock returns have been extensively investigated over the last century. Various model based approaches have been developed for measuring and forecasting important key factors based on the distribution of stock returns that affect stock trading. Due to the uncertainty in the occurrences of one–off market events, these models under or over estimate the actuarial market risk and return. Fundamental characteristics including the first four moments, co-moments and autocorrelation in the distribution of stock returns have received much attention because of their importance to various investment analyses including asset pricing theories, examining market efficiency and risk estimation. There are a number of inconsistent findings and arguments associated with these characteristics and this chapter discusses several selected issues.

Shewhart principles have been commonly used in industrial applications focussing not only on the quality of the product or service, but also for understanding process variation. This study will investigate the use of Shewhart principles for modelling the behaviour of stock returns. In particular, a set of propositions is put forward based on the Shewhart philosophical view to justify the appropriateness of the proposed approach.

New statistical methods are developed to investigate the identified research issues and these propositions.

1.1 Motivation

Stock prices change within very short time intervals with thousands of trades being made every second. Demand and supply theories, company financial information (earnings, stock splits, revenue, debt), news (whatever investors feel appropriate) and investor sentiment are the main causes of stock price variations. Investors have different expectations, and information dissemination through the market is not consistent. Financial decisions and strategies are always based on the formal and informal information arriving over time. Stock prices therefore may not fully reflect the information flowing into the market. This is an ever evolving process. As a consequence, the investigation of market behaviour under varying circumstances is important for financial analysts and researchers.

This study mainly focuses on the dynamics of stock returns including the first four moments, co-moments and autocorrelations in stock returns. It has been noted that the expected relationships and properties in stock return distribution under standard or expected behaviours of the market cannot always be justified by empirical research findings. This study has identified three main issues listed below.

Issue 1 : Is there always a positive risk–return trade–off?

> The mean-variance portfolio analysis of Markowitz (1952) gives a framework to manage a portfolio of assets such that the expected return is maximised for a given level of risk. Investors will have different risk preferences. The level of risk is the variability in returns that an investor is willing to accept in their portfolio. If the investor knows the level of risk that they are willing to accept, then the risk–return

trade–off determines how appropriate assets can be included in their portfolio to maximise the return.

In practice, stock returns are not normally distributed; therefore, the popular mean-variance analysis has become controversial. For example, Sheikh and Qiao (2010), Kim et al. (2015) reported dissatisfaction with mean-variance analysis because the model assumes normality and uses the variance as a risk measure. Among many others, Ghysels et al. (2005); Guo and Whitelaw (2006); Ludvigson and Ng (2007); Kanas (2012); Bollerslev et al. (2013); Kinnunen (2014) support a positive risk–return trade–off. On the other hand, some of the empirical literature (Kim et al., 1993; Campbell and Hentschel, 1992; French et al., 1987; Bollerslev and Hao, 2006; Lettau and Ludvigson, 2010; Ghysels et al., 2014) found a negative relationship. For example, according to French et al. (1987, p.27), "There is also a strong negative relation between the unpredictable component of stock market volatility and excess holding period returns", Wu (2001, p.837) noted that "negative (positive) returns are generally associated with upward (downward) revisions of the conditional volatility". Ghysels et al. (2014); Kinnunen (2014); Salvador et al. (2014); Wu and Lee (2015), showed that the risk–return relationship varies through time. The simulated learning model, described in Calvet and Fisher (2007), also explained the impact of information quality on the skewness/kurtosis trade–off and its link to varying volatility. Little attention has been given to the skewness/kurtosis trade–off in asset pricing theories. Inconclusive evidence for the mean/standard deviation trade–off and the importance of skewness/kurtosis trade–off are therefore worth investigating.

Issue 2 : Are co-moments measured accurately within extensions to the two-moment capital asset pricing model?

The capital asset pricing model (CAPM) can be used to calculate the required rate of return for an asset when adding or removing an asset from a well diversified portfolio. Sharpe (1964) and Lintner (1965) transformed the basis of Markowitz (1952) portfolio theory (mean–variance analysis) by assuming a utility function which is based on the first two moments. This implies that the model assumes the probability distribution of stock returns to be symmetric or completely described by the first two moments.

Due to non-normality in the distribution of returns, both skewness and kurtosis have been included in some of the later capital asset pricing models. Investors are interested in the association between the returns of an asset to a portfolio's skewness and kurtosis; therefore, co-skewness and co-kurtosis measures have been proposed in Rubinstein (1973); Kraus and Litzenberger (1976); Friend and Westerfield (1980); Harvey and Siddique (2000); Dittmar (2002); Jondeau and Rockinger (2006); Ang et al. (2006); Post et al. (2008); Harvey et al. (2010). The two–moment (mean–variance) model has therefore been extended to the first three-moments and eventually into four-moment case (mean–variance–skewness–kurtosis).

Hung et al. (2014) compared the effect of co-moments over crisis and non-crisis periods and revealed that practitioners may accept co-moments during crisis periods. All these studies claimed that the mean-variance CAPM overestimates expected returns relative to true market risk because it ignores the impact of skewness and kurtosis.

Issue 3 : How does autocorrelation in stock returns affect market efficiency?

Numerous authors have presented claims over the last forty years on why autocorrelation is observed in returns series. Scholes and Williams (1977), Boudoukh et al. (1994) and Campbell et al. (1997) claimed that positive autocorrelations are caused by non-synchronous security trading which tends to create spurious autocorrelations in individual series. However, Llorente et al. (2002) found that autocorrelation values at the first lag of daily returns are negative for small volume stocks with a large bid-ask spread, and positive, but very small, for large volume stocks with a small bid-ask spread. In contrast, Anderson et al. (2013) noted that partial price adjustments are a major source of autocorrelation and genuine autocorrelation can arise due to behaviours such as bid-ask bounce while Lim et al. (2008) argued that the autocorrelation in stock returns is all spurious. Autocorrelation created by only price generating process known as 'true autocorrelation' and else 'induced'. Stock return models (see, for example So et al., 2007; Polasek and Ren, 2001) rarely separate induced autocorrelation from true autocorrelation. The unresolved issue is to identify the strength of the "true" autocorrelation from the autocorrelation which is "induced".

One of the biggest debates in finance is whether the stock market is *efficient*; for example, Lo and MacKinlay (1988), Malkiel (2003) and Malkiel (2005) argue that the evidence of apparent anomalies and bubbles contradicts the efficient market hypothesis. Investors' biased behaviour was viewed by Shiller (2005) as responsible for market inefficiency (known as *irrational market behaviour*). An important implication of an efficient market hypothesis is that stock prices approximately follow a random walk model (see Fama, 1965, p.56). Also, the efficient market hypothesis itself implies that successive price changes are independent (see Fama, 1970, p.386). By considering all these facts, autocorrelation in stock returns is become an important statistical property for discussing market efficiency.

All of these issues are important for market participants when making investment decisions. There is sufficient evidence to prove that the effects of abnormal market fluctuations can not be explained by standard theories, see Reinganum (1981); Fama and French (1992); Malkiel (2003). For example, debt and mortgage–backed assets in the United States (US) were the underlying causes of the global financial crisis of 2007–2008 which resulted in a huge market threat to large financial institutions, national governments and to markets around the world. Value–relevant events such as the September 11, 2001 terrorist attack affected not only the national but also international economic environment.

Abnormal returns reflect extraordinary, infrequently occurring events or shocks that have important effects on modelling and predicting market risk. Financial data is clouded by larger variations caused by the unforeseen market events. The basic statistical problem is that analysis becomes more complex and risk estimators become irrelevant and incorrectly represent the true risk. McSweeney (2009, p.836) has noted that “Extensive and often significant unpredictable and unanticipated events occur. So, errors in valuations, which are based on expectations, will also be extensive and significant". Investors are interested in when and how the periods with and without stock return fluctuations can be analysed and to what extent they should be concerned with infrequent market events.

In the current context, alternative statistical methods are available to detect changes relative to a fitted model and to divide different components of the returns. Fitted models account for the larger sources of variability, hence become complex; and not sensitive to small but significant changes in the process. Applying quality control techniques to financial markets was suggested several decades ago by Roberts (1959). He states “a financial analyst should find much of interest and relevance in methods of quality control" (see Roberts, 1959, p.8). Modified Shewhart control chart procedures have been suggested in Severin and Schmid (1999), Kramer and Schmid (2000), Schipper and Schmid (2001a), Golosnoy et al. (2010), Schipper and Schmid (2001b). In particular, Andreou and Ghysels (2006), Bock (2008) and Garthoff et al. (2014b) used control charts to detect significant changes in stock return series relative to the fitted time series model based on historical data. All of the studies were proposed to detect the changes in the mean or variance in the underlying target process. In these studies control limits were adjusted according to the fitted time series models or the residuals were used as a control variable. The issues of modelling unpredictable events however, remains challenging and unresolved.

This study proposes Shewhart methodology Shewhart (1931) to discuss the inconclusive empirical results found in the literature referred to earlier which contradicts the standard theories. Quality management tools are used in the industrial sector across a very wide range of applications, hence this alternative approach will have a potential contribution well beyond empirical finance studies. Given that stock returns display variability (resulting from fundamentals, information, and market expectations) and understanding this observable variability continues to be the basis for many studies, there is a real need to consider the contribution Shewhart methodology can bring to this area of research. In the next section, three new propositions are presented. These propositions have been constructed by integrating the statistical features in stock returns which are logically equivalent to the ideas behind Shewhart's principles.

1.2 Shewhart postulates applied to financial data

Following the suggestions in the past to use quality management principles in financial return series, the Shewhart's principles were used by Govindaraju and Godfrey (2011). Walter Shewhart (Shewhart, 1931) introduced this simple, but effective methodology in the 1920s for assuring economic control of a process based on sound scientific reasoning. Even though his goal was to have statistical theory serve the needs of industry, Shewhart methodology has been widely used in the service sector and provides a way of looking at any process that displays variability. He proposed that the variation in any process can be attributed to either common or special causes. Common cause variation is the underlying variation that is always present in a process, while special cause variation is due to events that are more "one-off" in nature.

Every process is subject to a certain amount of natural variability caused by different types of input variables. Shewhart (1931) named this inherent variability "constant system of chance causes", whereas Deming (1986) named it "common causes". Security markets are always subjected to a certain amount of variability caused by noise-traders and other frictional price changes. These have been called "market noise" by Black (1986), "normal innovations" by Maheu and McCurdy (2004), "the efficient return components" by Bandi and Russell (2006) and "permanent innovations" by Shively (2007) to name just a few. Financial professionals believe that the existence of this natural variability in financial markets is a natural consequence of the activities that create the liquidity necessary for proper functioning of the market (Black, 1986).

If the variability in an industrial process is due to detectable causes such as an inappropriate batch of raw material, faulty equipment, etc., then the process will be operating in an unacceptable fashion. Such sources of variability are called "assignable" (Shewhart's terminology) or "special" (Deming's terminology) causes of variation. The variation in stock returns that Shewhart theory would consider to be "assignable" can be linked to a number of ideas found in the finance literature. Examples of terms used for return anomalies are "irregularities or deviations from common or natural order or exceptional conditions" by Frankfurtera and McGoun (2001); "jump innovations" by Maheu and McCurdy (2004); "microstructure noise" by Bandi and Russell (2006); "outliers" by Charlesa and Darné (2006); and "temporary innovations" by Shively (2007).

Shewhart principles give a protection compared to parametric modelling approaches when dealing with unusual observations. In presence of a large number of outliers in return series and their uncertainty result in over complicating the existing models. It is always questionable and create additional source of risk how or to what extent these models accurate and appropriate. Shewhart framework emphasizes the requirements of dealing special causes separately.

Shewhart (1931, p.266) presented the following three postulates on the common causes of variation from an engineer's point of view.

1. All chance systems of causes are not alike in the sense that they enable us to predict the future in terms of the past.

2. Systems of chance causes do exist in nature such that we can predict the future in terms of the past even though the causes be unknown. Such a system of chance causes is termed constant.

3. It is physically possible to find and eliminate chance causes of variation not belonging to a constant system.

A set of propositions asserted in this study is defined under Shewhart's notion of common and special cause variations and retains the philosophical view of Shewhart postulates. The research issues described in Section 1.1 are linked to these propositions which are the underlying assumptions for many models in finance.

> At the very beginning of empirical finance studies, it was assumed that stock prices behave according to the random walk model. By using the central limit theorem, it is therefore assumed that stock returns would be normally distributed.
>
> In later years, many authors, including, Scott and Horvath (1980) and Peiró (1999) showed that financial returns exhibit stylized features, including excess skewness and kurtosis. The asymmetry (third moment) and the thickened tails (fourth moment) of the returns distribution are primary properties of interest even under rational market behaviour.
>
> The departure from normality observed in the empirical data led to the family of stable Paretian distributions for the stock returns, (see Mandelbrot, 1963). Various parametric models for stock returns distribution have been summarised in Cont (2001). In addition, Mills (1995) examined the shape of returns distribution by using Tukey's g- and h-distributions, but, finally concluded that the distribution is dependent on the inclusion of data in some specific time period with huge market

fluctuations, for example, Black Monday in October 19, 1987. After considering this evidence, the following proposition is constructed for the univariate case under Shewhart framework:

Proposition 1 : The distribution of stock returns will follow a normal distribution under common cause variation.

In addition, several asset pricing and risk estimation models also assume that multiple stock returns series follow a multivariate normal distribution. Richardson and Smith (1993), Ané and Labidi (2004) and the references therein provide evidence for the lack of multivariate normality in stock returns. The univariate scenario (*Proposition* 1) is extended to the bi-variate situation in the following proposition.

Proposition 2 : The joint distribution of asset return and the market return will follow a bi-variate normal distribution under common cause variation.

The weak-form of the efficient market hypothesis argues that the current price of an asset incorporates the past prices information. So, price series approximately follow a random walk model and hence stock returns become a white noise series (see Al-Loughani and Chappell, 1997, p.174). Many authors including Khan and Vieito (2012), have used autocorrelation in stock returns to examine market efficiency (details are given in Chapter 8). Outliers are often observe in a series of stock returns and their presence can alter the true autocorrelation structure, Chernick et al. (1982). The following proposition is therefore put forward based on the Shewhart philosophical view:

Proposition 3 : Stock returns will show zero autocorrelation in the common cause periods.

1.3 Objectives & thesis structure

The goal of this study is to make a simple and useful contribution to the tools available to investors. To fulfil that goal, the three main objectives of this study are to

- Investigate the appropriateness of the Shewhart approach for assessing the behaviour of stock returns.
- Demonstrate that this approach will help to understand the contradictory arguments and empirical findings published in the financial literature.
- Show that Shewhart's principles can be used to understand the behaviour of stock returns.

The work is presented in nine further chapters. Chapter 2 reviews the properties of stock returns and the applicability of quality management principles. The use of Shewhart's principles to investigate stock market behaviour is discussed using the basic actions found in an industrial process and financial markets.

Chapter 3 presents the control chart techniques used to separate the special and common causes for a financial series. Control limits for the Shewhart S chart are used to separate common and special cause periods. Robust estimation methods for calculating subgroup standard deviations are also presented. Robust estimators for σ and robust chart procedures are applied for financial data. A procedure is also given for partitioning individual observations. This chapter demonstrates the common and special cause separation using real data; behaviour of the first four moments in partitioned data; and the effect of subgroup size. All these methods are implemented using R statistical software (R Core Team, 2015) in the new the "QCCTS: Quality Control Charts for Time Series" R-package developed for this study.

Chapter 4 investigates the *Proposition* 1 using the first four moments in total, common and special cause periods. Then all the trade-offs among the first moments are considered to address *Issue* 1 identified in Section 1.2. In particular, mean/standard deviation and skewness/kurtosis trade–offs are investigated using both theoretical distributions and empirical data. Trading volumes in the partitioned data are also analysed in this chapter.

New control charts for paired data are presented in Chapter 5. In portfolio management, investors monitor stock return variations relative to the changes in the market or another asset. For this setting, the joint behaviour of returns is incorporated in partition data under Shewhart principles. Price quotes of a market index or a stock are always recorded at the same frequency of time; hence, data become available as paired observations. The joint behaviour of the returns series is considered using the control charts based on a difference series of paired data. The chapter starts with a review of control chart procedures for monitoring paired variables. A new chart based on S_d; the subgroup standard deviations of the differences, is proposed. The performances of difference charts are investigated using average run length properties for various out-of-control scenarios. Both parameter known and unknown cases are considered for Phase I analysis. Difference charts are also illustrated using a finance example.

Chapter 6 investigates the co-moments in the total and common cause periods for bi-variate data. The difference chart approach from Chapter 5 is used to partition bivariate data, for example returns from a selected asset and a market index. This chapter intents to contribute to resolving *Issue* 2 which is based on the extended CAPM, and investigate *Proposition* 2. By first principles, symmetric forms of measures are available for both co-skewness and co-kurtosis. The equivalence is therefore investigated for these measures in total and common cause periods. In addition, new alternative measures are defined for co-moments based on the differences of non-matching pairs of observations.

A new theoretical tool is investigated in Chapter 7 to examine the autocorrelation in stock returns. The main purpose is to develop a method to identify the true autocorrelation in stock returns in order to investigate *Proposition* 3 and *Issue* 3 in Chapter 8. An odd–even data splitting method is proposed to reduces the impact of autocorrelation in the complete series. This chapter focuses mainly on the autocorrelation properties of split series. Autoregressive and moving average parameters are identified for odd–even segments when the complete series is a linear time series.

Chapter 8 empirically investigates the autocorrelation in stock returns using the results derived in Chapter 7. Having consecutive stock prices in the definition, stock returns do show significant autocorrelation depending on the underlying price structure. In this chapter, known time series models are used to demonstrate this behaviour. Further, stock returns are possibly contaminated by investors feedback on different infrequent events. Hence, true autocorrelation in stock returns cannot be identified. The odd–even split proposed in Chapter 7 is employed to investigate both the effect of outliers and induced autocorrelation in stock returns in S&P 500 stocks.

Chapter 9 demonstrates selected applications of the proposed Shewhart approach in finance and other disciplines. Standard deviation, skewness, kurtosis and co-moments are commonly used as risk measures in asset pricing. In this context, it is argued that depending on an investor's preferences, risk measures based on common and special cause periods give more accurate and detailed advice to market participants. A summary of the study and suggestions for future research are given in the final chapter.

1.4 Data & software

The daily closing prices and trading volumes of S&P 500 companies listed as of July 1st, 2016 were retrieved from http://www.finance.yahoo.com. Average share price and market capitalization were obtained from http://www.cboe.com/products/snp500.aspx. A sample of data series and a table including company name, ticker symbol, average price and market capitalization used for analysis is provided in a repository at https://github.com/npremara/PhD_Data

The statistical analysis has been carried out using R statistical software (R Core Team, 2015) and the “xts" package (Ryan and Ulrich, 2013) to manipulate the time series data. The RStudio (RStudio Team, 2015) interface has been used to develop a new R-package to implement the Shewhart approach for financial data. The other dependency R-packages used are cited in the relevant chapters. In addition to the new R-package, R codes use for the simulation purposes are available in https://github.com/npremara/RCodes.

Chapter 2

Market behaviour and Shewhart's principles

Variations in industrial processes is largely caused by factors which can be controlled and adjusted to a large extent. In the stock market, however, variation is caused by the individualized responses of investors to available information and market activities. Whilst the sources of variation are different, their effects, however, are common for any system. In order to understand/reduce the excessive variation see in the financial markets, it is of fundamental importance to review how and why Shewhart's principles are appropriate. The content of this chapter has been published in Premarathna et al. (2016b).

2.1 Introduction

The empirical finance literature discusses rational market behaviour under the efficient market hypothesis and irrational market behaviour under behavioural finance theory. The temporal price movements show the reactions of the market participants to the information coming into the market. Over–reactions may occur according to the way that the market participants respond to the information and news. Sometimes, the market seems to react irrationally to economic or financial news, even if that news is likely to have no real effect on the fundamental value of stocks itself. By comparison, production processes are seen to be efficient when various factors, such as machines and raw materials, meet their required specifications and fail to give desired level of output when one or several of these factors fails. Financial markets and production processes have much in common in the way that we observe variability. The aim of this chapter is to discuss the suitability of Shewhart principles in understanding the behaviour of stock returns.

Stock prices frequently subject to price adjustment according to the fundamental economics principles. At the most basic level, the stock market works according to the laws of supply and demand. Any given trade reflects a price for which there is a willing buyer and a seller. The price in the marketplace will fluctuate until equilibrium between supply and demand is attained. Deming (1986) used the funnel experiment to demonstrate the negative effects of tampering with the system without understanding the actual variation occurring within the process. In this chapter, trading behaviour and the funnel experiment rules are compared which further confirms relevancy of the proposed approach.

The rest of the chapter is organized as follows: In Section 2.2, rational and irrational market behaviour are discussed. Section 2.3 reviews Deming's funnel experiment and market behaviour. In Section 2.4, the issues that differentiate production processes from the stock market are given. Section 2.5 summarises the chapter.

2.2 Rational and irrational market behaviour

Nobel laureate Eugene Fama stated "A market in which prices always 'fully reflect' available information is called 'efficient' " Fama (1970, p.383). Fama argued that individual investors form expectations rationally; markets accumulate information efficiently; and, equilibrium prices integrate all available information immediately. Market efficiency is considered with respect to an information set and three forms of information subsets have been discussed:

Weak–form Efficiency : Information is based on historical prices. The principle implication in the weak–form market efficiency test is that an investor cannot use past share price movements to predict new price movements in their attempts to earn abnormal returns. This also supports the argument that share price movements are totally unrelated to the provision of new information and as a result move randomly.

Semi–strong–form Efficiency : The information set includes all publicly available information. The semi–strong form states that the current market price reflects all publicly available information; no one can consistently outperform in the stock market and earn abnormal returns.

Strong–form Efficiency: The information set includes monopolistic access information. The strong form implies that current market price reflects all relevant information, whether publicly or privately held.

Furthermore, market friction is interpreted as anything interfering with trading. If the market is rational then market friction is created. We also need to clarify an important distinction between financial market frictions and market inefficiencies. We assume that asset prices reflect all available public information but not necessarily all private information. Pricing errors, if they exist, are not financial market frictions. Even if an asset is mispriced, market participants make their choices and weight their portfolios using this current incorrect price. The markets can be efficient yet have frictions that interfere with trade.

In the meantime, other scholars have been searching for an alternative theory that might explain the apparent anomalies. Fama's joint Nobel laureate, Robert J. Shiller provided evidence in the 1980s, on the importance of changes in both economic fundamentals and changes in opinion or psychology in speculating price movements. The attitude of investors is of great importance in determining the prices of speculative assets. Shiller (1993b) argued that the stock market was overvalued because some investors have unreasonable expectations that are not based on relevant research. From his point of view, investors believe that earnings will grow extraordinarily fast thereby producing the extraordinary profits necessary to sustain unrealistically high share prices; and in turn, this will lead to extraordinary price/earnings ratios. According to Shiller (2005) excess variability can therefore be expected in the presence of irrational market behaviour. Under the perspective of Shewhart 's theory, the source of variations which are accounted by rational and irrational market behaviour are therefore largely proportionate to common and special cause variations respectively.

On the other hand, even if all information is disseminated through the market, the stock market is always subject to noise, and there is uncertainty about future demand and supply conditions within and across sectors (see Black, 1986) or market friction. Noise is defined as unwanted sources of variation in any process. For the stock market, noise traders those who have "expectations or sentiment that are not fully justified by information" (see Shleifer and Summers, 1990), generate market noise and Black (1986) and Trueman (1988) have noted that noise traders play an important role by ensuring market liquidity. Noise is also expected for manufacturing processes because of unavoidable factors and thus is acceptable at a certain level. In a production process, noise is controlled after identifying its main source in order to maximise the performance whereas noise cannot be reduced by regulating the market.

2.3 Market behaviour and Deming's funnel experiment

This section establishes the validity of using quality improvement concepts to analyse stock market behaviour. In any kind of process, unnecessary interventions such as tool changes (production process) or speculation (stock market) should not become another source of special cause. We need to emphasize here, external adjustments to a process can be controlled in an industrial setting which is not always possible in the stock market due to risk taking human behaviour. Suppose an event such as credit market turbulence has increased the trader's expectation of volatility in the stock market. The impact of such an event could change the investors' trading strategies (hesitation to buy and eagerness to sell). As a result, stock prices drop to balance buying and selling volumes. Consequently overreaction to market information leads to increased volatility and over adjustment of prices.

Over(under) reaction to market information leads to increased (decreased) volatility (variation over time) and over(under) valuing stocks. This behaviour is somewhat equivalent to the tampering described by Deming (1986) in his funnel experiment. The funnel experiments devised by Deming (1986) explained the adverse effects of making adjustments without understanding the nature of variability permitted by common causes. In Deming's experiment, a funnel is mounted on a stand and the spout is adjusted toward a target. A marble is repeatedly dropped through the funnel and the final resting position for each drop is noted. Deming (1986) demonstrated how decisions about moving the funnel could lead to unexpected variability of outcomes. In a financial context, the target for each market participant would be achievement of the true market value of an asset. Market prices fluctuate (like marble drops) as individual investment decisions are made. What happens behind the scenes in price adjustments apart from the fundamentals cannot be clearly identified in the financial market because there is always an opposite position in the market to make trading activities possible. In order to understand the impact of the decisions made by rational and irrational investors (see Section 2.2), we now demonstrate how the adjustment rules that Deming formulated, are consistent with specific trading actions and market conditions.

Rule 1: *The funnel remains fixed, aimed at the target.* This rule will produce a stable distribution of points with the minimum variability that exists in any process. There will always be variation (different spots where the marbles come to rest) in the system. In the context of the stock market, the target is the true market value of a certain asset and rational (efficient market) condition is equivalent to a fixed position of the funnel. Normal returns on a security are expected to be stable over time. Hence, the market is efficient relative to all the available and relevant information; variation that exists in the market can be considered to be noise (common cause variations). Even though price adjustments are continuously happening over time, if the market is efficient, only random variation can be expected. Investors are not always able to obtain all the information they would need to make the best possible decisions, so market prices are not reflective of the true underlying market value. Such occurrences can be explained through Rule 4 in Deming's funnel experiment.

Rule 4: *Move the funnel to the position where the last marble dropped.* In this scenario investors believe that the current price correctly represents the true market value implying an efficient market condition. Markets are driven by more than fundamentals, and investor psychology plays a huge role and helps explain why asset prices go through booms and busts. Investors may not always be rational, for example: extrapolating the present into the future, giving more weight to recent spectacular or personal experiences, overconfidence, slow response in adjusting expectations and selective use of information etc. Financial market behaviour is different from Deming's funnel experiment under Rule 1 and Rule 4 due to the crowd behaviour of investors rather than each investor acting as an individual. Irrational market behaviour as described by Shiller, leads to "bubbles and busts" in stock returns, the magnitude of the shift provides pricing errors, and as a consequence price adjustments create excess volatility.

Rule 2 and Rule 3 described in the funnel experiment are largely related to individual market behaviour in the stock market. They could be considered as different types of market speculation activities. Market speculation has played a major role since the beginning of modern stock exchanges. There are different types of stock market speculators, each with their own view of how to capitalize from price fluctuations in the stock market.

Rule 2: *Move the funnel from its previous position a distance equal to the current error (location of drop), in the opposite direction.* Rule 2 produces stable output, but increases the variation compared to Rule 1. In the stock market there is always an opposite trade position to make trading activities possible. Therefore, the applicability of this rule can be described through *uptick* and *Downtick* financial transactions: *uptick*: An uptick has occurred if a stock's price has increased in relation to the "last tick" or trade. *Downtick*: In order for a downtick to occur, a transaction price must be followed by a decreased transaction price. In Deming's funnel rule, the next adjustment to the process based on the previous position and similar relationship can be viewed in the occurrences of *uptick* and *Downtick* transactions.

Rule 3: *Move the funnel to a position that is exactly opposite the point where the last marble dropped, relative to the target.* The system will break out, as the marble will eventually move away in the opposite direction from the target. "Short selling: a short sale is the sale of a security that is not owned by the seller, but that is promised to be delivered". A market transaction in which an investor sells borrowed securities in anticipation of a price decline and is required to return an equal number of shares at some point in the future; the outcome of a short sale is the opposite of a regular buy transaction.

It should be noted that, in stock market transactions, there may be a number of rules other than these four situations exercised by individuals and groups of people at the same time. The activities leading to over-adjustment of stock prices cannot directly link to the rules described in Deming's funnel experiment because there is a number of physical and human activities lead to price adjustments. Unnecessary adjustments (market behaviour) do create excess variability (volatility) within the system and trade-offs among the distributional properties in the stock returns distribution can therefore be expected. This study observes and provides an explanation of such patterns through an empirical data analysis in later chapters.

2.4 Industrial process vs. stock market

Both common and special causes create variations; common causes contribute to "controlled" variation while special causes contribute to "uncontrolled" variation. Shewhart (1931, p. 6) defined *controlled variation*: "a phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to vary in the future. Here it is understood that prediction within limits means that we can state, at least approximately, the probability that the observed phenomenon will fall within given limits." In the stock market, irregularities occur because investors either overreact or only partially adjust to information as it comes into the market. The information on variation which has been experienced in the past such as stock splits, dividends, earnings, index reconstitution announcements etc., might enable a certain level of prediction. However, we cannot explicitly state that unexpected market events lead to "uncontrolled variation" every time. According to Deming (1986, pp. 321-322), after removal of all the detected special causes (meaning the process has achieved statistical control), improvements in the system are then the responsibility of management. But, with respect to a financial market, who are the managers? Stock prices are frequently subject to price adjustment that should be based on the fundamental principles of economics. Having said that, at the most basic level, the stock market works according to the laws of supply and demand; every given trade reflects a price where there is both a willing buyer and a willing seller. The price in the marketplace will fluctuate until equilibrium between supply and demand is reached. There is therefore a combination of forces that affect the price of a given stock with the role of management being filled by a large number of market participants that have differing views of the economic value of any new information.

It needs to be emphasized that the following issues differentiate an engineering process from a stock return process. In a production process, it is possible to eliminate the effect of special causes, but not in the stock market. Always, price adjustments are made according to the will and wisdom of the investors over time and response times are not always instantaneous. Beliefs, speculation and perceptions do not have any effect on the result in a manufacturing process. Investors react differently to particular news. For example, the arrival of new messages about the spreading of cattle disease which could have an impact on the dairy product related stocks. Hence, that becomes a special cause incident for a particular group of investors. In industrial settings, quality is inversely proportional to excess variability (often thought of as unwanted or harmful) which needs to be removed. It could be as a result of a technical problem which might lead to replacement of equipment for example. In financial markets, however, excess volatility (stock price volatility exceeding that justified by fundamentals, see Shiller, 1993a) cannot be removed because it occurs as a result of unexpected information coming into the market which potentially leads to partial or over adjustment in stock prices.

Price adjustments largely depend on the decisions of investors, financial institutions and other government bodies, and price correction is dependent on the extent to which the current market price has already incorporated the new information. It is to be noted that the common practice of using returns (the difference in prices) means that a shift in price caused by new information is reflected as a one-off event in the returns series, while static price is reflected as a zero return. Changes in volatility influence the distribution of portfolio returns and assessment of portfolio risk and therefore play an important role in managing risk, selection of portfolios, and in pricing derivatives. For example, in response to unusual or unexpected events, market volatility could increase which might lead to incorrect investment decisions (see Govindaraju and Godfrey, 2011).

In a manufacturing process, control charts use to identify the special causes and then remove the effect of them updating or removing the cause factors. In contrast, in the financial market, we cannot reduce or remove the effect, it not completely under control of any party and depend on the investors motives. It is important to state that, therefore, control chart procedures for manufacturing process cannot be implemented for the stock market in exactly the same way, but market participants could use the framework we have suggested in this work as a basis for their future investment decisions.

2.5 Summary

Shewhart's principles are often used in industrial applications but their applications are not common in the empirical finance literature. This study reviewed how the Shewhart philosophy could be related to understanding the variation in stock returns. In this chapter, the nature of variations in the stock market exemplified using Deming's funnel experiment to compare these variations to those arising in an industrial process. A few but important financial topics were chosen to demonstrate the similarities between the two contexts including market efficiency vs common cause variation, rational market behaviour vs common cause variation and irrational market behaviour vs special cause variation. Issues arising from the differences in the mechanisms of the two contexts were recognised and have been addressed. Overall, this discussion justifies that the quality management principles embodied in the Shewhart philosophy are not only appropriate for the analysis of stock returns but provide further opportunities to understand and explain variations.

Chapter 3

Control chart techniques for financial applications

To demonstrate the application of Shewhart principles to financial data, we need a criterion to separate the variation due to common and special causes in stock return series. In this chapter the control chart techniques that will be used in subsequent Chapters (4, 5, 6, 8 and 9) are presented. The limitations of the standard control chart methods for separating the common and special causes of variations in financial data are first explained. Robust estimation methods are shown to be more appropriate to volatility encountered in financial data. The partition of time series financial data into common and special cause periods is achieved using subgroups as well as individual observations.

A new R-package: “QCCTS: Quality Control Charts for Time Series" has been developed based on the methodology developed and presented in this chapter. This R-package is being made available at https://github.com/npremara/QCCTS. It is believed that new R-package can be used not only to financial data but for all other applications involving time-series data.

3.1 Introduction

This study is concerned with the empirical relevance of partitioning the observed variation in stock returns into two components based on Shewhart's framework. Stock returns exhibit occasional jumps which arise from various circumstances including company-specific, market-wide and macroeconomic information. The variations labelled as special causes are result of excess variability possibly due to unforeseen information becoming available to the market.

Control limits not only provide a means of purposive control over quality but also serve as a decision criterion for identifying what variation might be left to chance. Shewhart (1931) introduced the idea of "rational subgroups" to decompose the common and special causes. A rational subgroup is a sample of observations formed in a way that minimizes variation among the subgrouped observations, and maximizes the variation between the subgroups. In other words, the member units of a subgroup are as homogeneous as possible.

The term subgroup usually refers to a short series of data, for example the daily returns of an asset pertaining to a week. For each subgroup, the mean (return) and the standard deviation (risk) can be used as control statistics. Both statistics are important to investors as they try to minimize risk for a given amount of return or in complementary fashion, maximize return for a given level of risk. Events including public holidays and market closures that alter the number of observations in each subgroup (week/fortnight/month etc.) must also be accounted. In order to identify the periods of special cause variation, the decision rule used in Govindaraju and Godfrey (2011) is generalised to accommodate varying subgroup sizes.

In Section 3.2, a procedure based on the Shewhart *S* chart to partition the common and special cause periods is explained. The basic idea of separation of common and special causes is illustrated using an example. In Chapter 1, the importance of the first four moments of the distribution of stock returns was emphasised. Mean, standard deviation, skewness and kurtosis are therefore examined in partitioned return series. This procedure will be employed in Chapter 4.

Having established the applicability of the proposed method, the skill is then to select the appropriate size of the rational subgroup; we contend that this is actually dependent on individuals' motives for participating in the market or the frequency that investors observe the market while investing. For example, short-term traders usually monitor the market on a daily or weekly basis but for long-term investors it could be monthly or quarterly. Even though high frequency financial data are available we should focus on the data selection procedures suitable for market observation behaviour of the investors (Leinweber, 2007). This study restricts to illustrations using weekly/fortnight subgroups. Garcia et al. (1992) has argued that the standard deviations over different definitions of one year periods are 'unstable'. This instability occurs as the variance estimates contain noise, but does not depend on how much historical data we consider to estimate these measures. It is shown that the estimates of volatility presented in their work are contaminated by short time periods within the year of data considered because these periods were indeed highly probable special cause periods. Standard deviation in common cause periods is then compared for different sizes of subgroups.

According to Rocke (1989, p.173), "The presence of outliers tends to reduce the sensitivity of control-charting procedures because the control limits become stretched so that the detection of the outliers themselves becomes more difficult". The impact of the outliers to be one of the most important aspects for financial data (Balke and Fomby, 1994). As a consequences, robust estimators are used to reduce the error in estimation for the unknown parameters (μ and σ) arising from the disturbances between and within subgroups (see Tatum, 1997). Schoonhoven and Does (2012) proposed a robust estimator which is based on the trimmed mean of subgroup interquartile ranges ($\overline{IQR}_{10}$) for σ and have shown that their estimator performs equally or better than the other existing robust estimators when there are 'diffuse disturbances'. The partition of common and special cause variations using an unbiased estimator for σ from $\overline{IQR}_{10}$ is therefore investigated along with the traditional Shewhart S chart criterion.

In standard control chart applications, two phases are involved (see Montgomery, 2011, p.198). In Phase I, trial control limits are calculated using a retrospective large sample when parameters are unknown. Many authors, including Jensen et al. (2006); Chakraborti (2007); Braun and Park (2008); Faraz et al. (2015) have examined the effect of estimated parameters on control limits for various control charts. Phase I estimation methods for other applications differ as discussed in Jensen et al. (2006). Other approaches for estimation σ which avoid the bias in estimation, for example, the square root of the pooled variance suggested in Saleh et al. (2015) should be considered. For some applications, such as monitoring stock returns, we expect high contamination due to special causes. Robust estimation procedures can be used for Phase I data analysis when contamination is high. Two robust chart procedures proposed by Nazir et al. (2014a) and Nazir et al. (2014b) using $\overline{TM}_{10}$ (a robust estimator for μ based on the trimmed mean of the subgroup trimeans) and $\overline{IQR}_{10}$ were employed for Shewhart $\bar{X}$ and S charts to calculate the unknown μ and σ for Phase II analysis. The algorithm first screens the subgroups and then the individual outliers are removed in the remaining subgroups. These two algorithms are summarised in Section 3.3 and will be employed in Chapters 5 and 6.

In Chapter 8, the autocorrelation in stock returns will be investigated. The identification of common and special cause periods therefore needs to be on the basis of individual observations to preserve and study the autocorrelation in daily returns. Section 3.4 gives the Shewhart chart procedure for individual observations which will be employed in Chapter 8.

In this study, Shewhart principles are applied to stock returns but this approach is valid for any type of financial series such as stock index, exchange rates and so on. An R-package would make it easy for practitioners to apply these techniques. The published R-packages (Scrucca (2004); Santos-Fernández (2013); Recchia et al. (2014)) available for control charts are not dedicated to time series data. The QCCTS-R-package has been specifically developed for designing Shewhart $\bar{X}$ and S charts with time series data. In Section 3.5, the main functions and features of the QCCTS-R-package are given. Section 3.6 concludes the chapter.

3.2 Shewhart control charts

In industrial settings where quality management is the main reason for employing the Shewhart methodology, subgroups displaying special cause variability are removed from the data series as the problem leading to the special cause is rectified. In a financial setting there is no guarantee that all special causes are independent and that these are able to be rectified by intervention. Rather, it is the market itself that will rectify any mispricing of the asset which may in turn lead to what might also be described as another special cause. Once a historical return series is obtained, we can employ existing control chart techniques to partition the common and special cause variations.

Let $X_{ij}, i = 1, 2, \ldots, m$ & $j = 1, 2, \ldots, n_i$, be the set of available observations considered to comprise the appropriate rational subgroups where m and n_i are the number of subgroups and the subgroup size respectively. Any subgroups with only one observation have been omitted from the analysis so in total there are N return values available for each stock. The sample standard deviation of the ith subgroup is given by,

$$s_i = \sqrt{\frac{1}{n_i - 1} \sum_{j=1}^{n_i} \left(X_{ij} - \bar{X}_i \right)^2} \tag{3.1}$$

The effect of varying subgroup size on the ith subgroup sample standard deviation s_i is corrected using the corresponding unbiasing coefficient $c_4(n_i)$ and it is given by $\bar{s}_i = \frac{s_i}{c_4(n_i)}$, where $i = 1, 2, 3, ...m$. In order to avoid substantial change in average within a subgroup due to the small fraction of anomalous measurements with an abnormally large deviation, an $\alpha = 5\%$ trimmed average standard deviation was employed (see Ryan, 1997). The trimmed mean of the subgroup standard deviation is given by,

$$\bar{S}_\alpha = \frac{1}{m - 2\lceil m\alpha \rceil} \left[\sum_{i=\lceil m\alpha \rceil + 1}^{m - \lceil k\alpha \rceil} \bar{s}_{(i)} \right] \tag{3.2}$$

where α denotes the percentage of subgroup data to be trimmed, $\lceil v \rceil$ denotes the ceiling function and $\bar{s}_i$ denotes the ith subgroup sample standard deviation.

The criterion for determining which variation must be left to common causes was based on the following procedure. Subgroups subject to special causes can be identified, using coefficients B_4 and c_4 to scale $\bar{S}_{\alpha=0.05}$ (here the unbiasing coefficient B_4 is a function of c_4 and therefore n_i) to give the upper control limit for series that are responding to common cause variation. Any rational subgroup having a within a subgroup standard deviation exceeding $B_4\bar{S}_{\alpha=0.05}$ will be then identified as a time period when some special cause of variation was causing the volatility in the process. This subgroup should be removed from the calculation of $\bar{S}_{\alpha=0.05}$ as it is contaminated by the special cause variation. There is then a need to re-calculate the control limit using the new $\bar{S}_{\alpha=0.05}$; several iterations may be needed to get an upper control limit that is based solely on subgroups whose variation is invariably based on common causes. However, a problem in the revision of control limits was encountered. As the upper control limit changed, more and more subgroups exceeded the new limit. It is clear that many of these must be false alarms. To counteract this problem, the following rule was used for terminating the chart revision process. The probability of an estimated average subgroup standard deviation falling outside the control limits was considered as 0.0027. If we have m subgroups, we expect $0.0027m$ subgroups to be identified as special causes. If the number of flagged subgroups is below the Poisson upper control limit of $0.0027m + 3\sqrt{0.0027m}$, we stopped the subgroup removal process. This method was used in Premarathna et al. (2016b).

3.2.1 An example

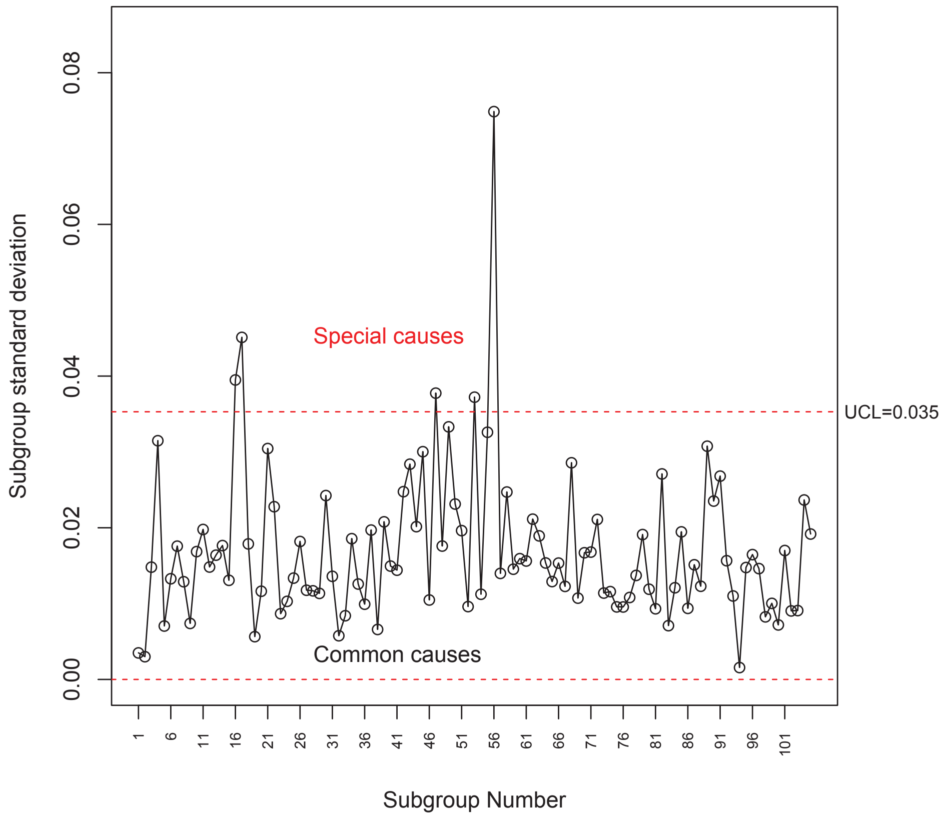
Remembering that this study is concerned with the empirical relevance of partitioning the observed variations in stock returns into two components based on Shewhart's framework, the Shewhart S chart for Apple Inc. returns was prepared. Figure 3.1 illustrates the partition of subgroups into common and special cause variations using the procedure given above.

Although we have used weekly subgroups for this illustration, it is believed that appropriate selection of m will be specific to the user. Intra-day data series rather than daily data may also prove to be a useful enhancement; having twice-daily records and keeping the time-horizon constant would double the subgroup size for example. A range of S&P 500 stocks was used for identifying common and special cause variations. Ten stocks were selected in different sectors during January, 2001 to December, 2015. Table 3.1 shows the standard deviation in the common cause periods for different subgroup sizes for selected stocks. Special care need for the stocks which show the increasing common cause standard deviation when the subgroup size increases, for example Apple Inc. and Priceline.com Inc. stocks.

Let us consider the distribution of the returns over time in order to investigate the partitioned data. Table 3.2 gives summary statistics for fifteen selected stocks. Series and ticker symbols are given in the first two columns. The other four columns give the mean, standard deviation, skewness and kurtosis respectively. However, significance tests for the excess skewness and kurtosis will be performed in Chapter 4 (Section 4.2).

Table 3.3 shows the mean, standard deviation, skewness and kurtosis for the daily returns after they have been separated into the subgroups that would be deemed to

Figure 3.1: S chart for Apple Inc. based on weekly subgroups of daily log returns-Phase I analysis



be driven by common and special cause variations. Even with limited sweeps over the subgroups to remove those with excess volatility, we see that these periods are having a much greater impact on the tendency for non–normality in the returns distributions for our selection of stocks. In particular, it should be noted that none of these stocks show skewness in the distribution of returns for periods when only common cause variation is affecting the price movements and kurtosis for these time periods is considerably lower than for the time periods when special cause variation is having an impact. Given we have used a criterion based on standard deviation to separate time periods, it is no surprise

Table 3.1: Standard deviation σ in common cause periods based on the different subgroup sizes: W-week, M-month and Q-quarter

Series	Ticker	1W	2W	3W	1M	1.5M	2M	1Q
Apple Inc.	AAPL	0.0188	0.0186	0.0189	0.0187	0.0208	0.0209	0.0235
KB Home	KBH	0.0252	0.0252	0.0243	0.0241	0.0243	0.0256	0.0285
Helmerich & Payne	HP	0.0216	0.0212	0.0211	0.0209	0.0221	0.0210	0.0248
Magna Inter.	MGA	0.0139	0.0142	0.0148	0.0146	0.0158	0.0165	0.0174
Johnson & Johnson	JNJ	0.0081	0.0082	0.0086	0.0088	0.0091	0.0093	0.0096
Citigroup Inc.	C	0.0137	0.0136	0.0162	0.0175	0.0193	0.0198	0.0197
Exxon Mobil Corp.	XOM	0.0116	0.0115	0.0116	0.0115	0.0120	0.0121	0.0134
BP plc	BP	0.0131	0.0128	0.0130	0.0129	0.0134	0.0136	0.0144
Microsoft	MSFT	0.0129	0.0129	0.0128	0.0134	0.0143	0.0155	0.0161
Priceline.com Inc	PCLN	0.0186	0.0183	0.0213	0.0231	0.0245	0.0271	0.0309

that the standard deviations are different, but we now see that there are differences in the mean returns for time periods of common and special cause variations. It is concluded that most unpredictable events affecting the price of these stocks are having a negative impact on the daily returns.

Table 3.2: The first four sample moments for the return series of selected stocks

Series	Ticker	Mean	Std.Dev	Skew	Kurt
Apple Inc.	AAPL	0.000519	0.0413	-28.2487	1283.5658
AutoZone Inc	AZO	0.000870	0.0172	0.3031	14.9443
Bank of America Corp	BAC	-0.000271	0.0331	-2.6995	73.5923
BlackRock	BLK	0.000568	0.0223	0.1351	10.7854
Boeing Company	BA	0.000224	0.0188	-0.3309	9.5255
BP plc	BP	-0.000117	0.0181	-0.4327	13.8465
Chevron Corp.	CVX	0.000012	0.0197	-10.6751	374.5699
Chipotle Mexican Grill	CMG	0.000956	0.0259	-0.4787	14.5274
Cisco Systems	CSCO	-0.000054	0.0245	0.1490	12.7618
Citigroup Inc.	C	0.000006	0.0499	25.0970	1162.6476
The Coca Cola Company	KO	-0.000092	0.0167	-19.0763	798.2032
Ensco plc	ESV	-0.000215	0.0274	-0.2702	7.5759
Exxon Mobil Corp.	XOM	-0.000036	0.0190	-11.6629	418.5591
General Electric	GE	-0.000090	0.0195	0.0768	12.3493
Google Inc.	GOOG	0.000707	0.0241	-8.4467	265.8383
Helmerich & Payne	HP	0.000053	0.0299	-3.9915	91.2742
Hewlett-Packard	HPQ	-0.000249	0.0259	-4.7846	125.7416
Honda Motor	HMC	-0.000223	0.0299	-27.1175	1268.1248
International Bus. Machines	IBM	0.000128	0.0156	0.1455	9.7282
Intel Corp.	INTC	0.000027	0.0229	-0.2080	10.4645
Intuitive Surgical Inc.	ISRG	0.001115	0.0348	2.3672	47.5783
J.C.Penney	JCP	-0.000122	0.0301	-0.0098	8.2609
Johnson & Johnson	JNJ	0.000002	0.0162	-21.2466	911.5409
JPMorgan Chase & Co.	JPM	0.000108	0.0259	0.2714	16.9567
KB Home	KBH	-0.000248	0.0340	-1.8632	43.1649
Lowe's Cos.	LOW	0.000141	0.0257	-9.6735	265.5073
Magna Inter.	MGA	-0.000006	0.0260	-10.0547	269.9397
McDonald's Corp.	MCD	0.000334	0.0143	-0.1395	9.7931
Microsoft	MSFT	0.000065	0.0215	-7.6065	242.9519
NetFlix Inc.	NFLX	0.000561	0.0532	-15.7445	564.0626
Nvidia Corporation	NVDA	0.000025	0.0412	-3.9470	73.4495
PepsiCo Inc.	PEP	0.000187	0.0120	0.0048	17.2860
Petr	PBR	-0.000475	0.0349	-3.7883	78.5621
Priceline.com Inc	PCLN	0.001805	0.0485	12.9358	490.1741
Tesoro Petroleum Co.	TSO	0.000592	0.0374	-1.8361	36.4027
Toyota Motor	TM	0.000177	0.0173	-0.0925	10.7410
Wal-Mart Stores	WMT	0.000034	0.0135	0.0722	8.6630
The Walt Disney Company	DIS	0.000351	0.0191	-0.0575	11.7367
Wells Fargo	WFC	0.000002	0.0277	-3.5187	127.8641
Yahoo Inc.	YHOO	0.000044	0.0325	-2.2315	64.5653

Table 3.3: Distribution of returns after separation into common and special causes

Ticker	Common cause Mean	Std.dev	Skew	Kurt	Special cause Mean	Std.dev	Skew	Kurt
AAPL	0.00142	0.019	0.069	3.640	-0.00564	0.104	-13.621	245.398
AZO	0.00079	0.012	0.010	3.919	0.00133	0.034	0.222	6.210
BAC	0.00033	0.013	0.070	4.026	-0.00201	0.062	-1.555	23.718
BLK	0.00029	0.015	-0.072	3.797	0.00181	0.042	0.045	4.430
BA	0.00060	0.015	-0.101	4.134	-0.00256	0.037	-0.138	4.669
BP	0.00022	0.013	-0.020	3.838	-0.00282	0.039	-0.201	5.301
CVX	0.00033	0.013	-0.243	3.595	-0.00303	0.051	-6.337	86.997
CMG	0.00061	0.018	0.051	4.110	0.00268	0.048	-0.526	6.573
CSCO	0.00032	0.015	-0.090	3.986	-0.00146	0.045	0.209	5.130
C	0.00000	0.014	-0.065	4.092	0.00006	0.099	13.447	313.106
KO	0.00029	0.009	0.010	3.774	-0.00255	0.039	-10.699	190.412
ESV	0.00008	0.023	-0.151	3.932	-0.00317	0.056	-0.119	3.767
XOM	0.00023	0.012	-0.208	3.751	-0.00244	0.048	-6.858	97.749
GE	0.00021	0.012	0.113	4.028	-0.00119	0.036	0.127	4.938
GOOG	0.00092	0.014	-0.038	3.917	-0.00060	0.051	-5.547	82.684
HP	0.00074	0.022	-0.113	3.774	-0.00604	0.068	-3.141	33.008
HPQ	0.00012	0.016	-0.070	3.900	-0.00190	0.051	-3.433	46.770
HMC	0.00021	0.015	-0.061	3.667	-0.00355	0.078	-13.235	238.111
IBM	0.00027	0.011	-0.059	3.947	-0.00056	0.030	0.203	4.162
INTC	0.00018	0.016	-0.034	3.789	-0.00081	0.044	-0.126	4.432
ISRG	-0.00008	0.019	-0.066	4.259	0.00552	0.065	1.498	17.489
JCP	0.00021	0.023	0.100	3.910	-0.00210	0.057	0.035	4.070
JNJ	0.00032	0.008	0.078	3.703	-0.00160	0.035	-12.138	238.189
JPM	0.00010	0.014	-0.098	3.940	0.00013	0.049	0.192	6.089
KBH	-0.00008	0.025	0.097	3.718	-0.00134	0.068	-1.772	20.043
LOW	0.00053	0.016	0.156	3.806	-0.00253	0.060	-6.078	71.955
MGA	0.00063	0.014	-0.088	4.380	-0.00298	0.054	-6.335	82.950
MCD	0.00031	0.011	-0.001	3.708	0.00061	0.029	-0.159	4.234
MSFT	0.00024	0.013	0.042	4.186	-0.00107	0.046	-5.241	77.991
NFLX	0.00112	0.027	0.176	3.844	-0.00276	0.124	-8.596	132.636
NVDA	0.00030	0.025	0.069	3.848	-0.00121	0.082	-2.819	26.377
PEP	0.00033	0.009	-0.005	3.446	-0.00074	0.025	0.104	6.927
PBR	0.00043	0.024	-0.039	3.600	-0.00728	0.079	-2.626	25.323
PCLN	0.00098	0.019	0.160	3.862	0.00424	0.091	7.770	158.262
TSO	0.00102	0.026	-0.149	4.318	-0.00251	0.082	-1.311	12.626
TM	0.00023	0.014	0.028	3.433	-0.00026	0.037	-0.081	4.782
WMT	-0.00010	0.010	-0.093	3.695	0.00088	0.026	0.021	3.929
DIS	0.00034	0.013	-0.140	4.250	0.00044	0.034	-0.028	5.246
WFC	0.00021	0.011	-0.043	3.868	-0.00067	0.054	-2.024	38.634
YHOO	0.00033	0.019	-0.037	3.887	-0.00122	0.062	-1.540	23.973

3.3 Stepwise robust chart procedures

A robust estimator for subgroup standard deviation ($\overline{IQR}_{10}$) is given in subsection 3.3.1. The unbiased estimator for σ from $\overline{IQR}_{10}$ is then used for standard Shewhart S chart procedure that have been followed in Section 3.2.

The stepwise robust chart procedure for Shewhart S chart (Nazir et al., 2014b) is summarised in subsection 3.3.2. The stepwise robust chart procedure for Shewhart $\bar{X}$ chart by Nazir et al. (2014a) used both $\overline{IQR}_{10}$ and $\overline{TM}_{10}$. A summary of this procedure was given in subsection 3.3.3. These two procedures were designed for equal subgroup sizes but unequal subgroup sizes in financial data are common due to market closures, public holidays etc. Hence, in this study, the two robust estimators found in Nazir et al. (2014b) and Nazir et al. (2014a) were adopted for unequal subgroups using the appropriate unbiasing constants. These two procedures were used to estimate the unknown parameters μ and σ from limited Phase I data which are required to find the Phase II control limits.

3.3.1 Robust estimator for σ

The interquartile range of subgroup i is calculated as $IQR_i = Q_{i,3} - Q_{i,1}$, where $Q_{i,1} = x_{i,(a)}$ and $Q_{i,3} = x_{i,(b)}$, with $a = \lceil n_i/4 \rceil$, $b = n - a + 1$, and $x_{i,(m)}$ the m^{th}–order statistics of subgroup i. Each IQR_i is corrected by dividing its normalizing constant, $x_{\overline{IQR}_{10}}(n_i)$. These values are given in Table 1 of Nazir et al. (2014b). The 10% trimmed mean of the sample interquartile ranges is used as an unbiased estimate for σ:

$$\overline{IQR}_{10} = \frac{1}{k - 2(\lceil k/10 \rceil - 1)} \left[\sum_{m=\lceil k/10 \rceil}^{k-\lceil k/10 \rceil+1} IQR_{(m)} \right]$$

The partition of common and special cause variations was repeated for the same stocks returns discussed in subsection 3.2.1 in order to examine the first four moments in partitioned data. It is found that the use of this robust estimator does not make a large difference to the partitioning.

3.3.2 Estimation of σ for Phase II

This procedure involves two main steps namely screening of subgroups affected by parameter shifts and then the removal of individual outliers (signal points) in the remaining unaffected subgroups.

Step 1: Divide Phase I data of the differences $x_{ij}, i = 1,2,...,k$ and $j = 1,2,..,n_i$ into k subgroups of size n_i.

Step 2: Calculate $\overline{IQR}_{10}$ as given in subsection 3.3.1. The variable control limits for screening the subgroup shifts are as follows: $\widehat{UCL}_1 = U_1(n_i)\frac{\overline{IQR}_{10}}{x_{\overline{IQR}_{10}}(n_i)}$ and $\widehat{LCL}_1 = L_1(n_i)\frac{\overline{IQR}_{10}}{x_{\overline{IQR}_{10}}(n_i)}$, and U_1, L_1 and $d_{\overline{IQR}_{10}}(n_i)$ are tabulated in Table 1 of Nazir et al. (2014b).

Step 3: Exclude all the subgroups whose IQR/d_{IQR} falls outside the control limits stated above.

Step 4: The residuals of each of the individual observation in the remaining subgroups are calculated by subtracting the *trimean* of corresponding subgroup. $res_{x_{ij}} = x_{ij} - TM_i$, where, $TM_i = (Q_{i,1} + 2Q_{i,2} + Q_{i,3})/4$ and $Q_{i,2}$ is the median of subgroup i. The residual values of each observation are then screened. The average IQR of remaining subgroups $(\overline{IQR}')$ is used to identify the outliers in residuals. The control limits are varied depending on the subgroup size: $UCL_{ind} = 3\overline{IQR}'/x_{IQR}(n_i), LCL_{ind} = -3\overline{IQR}'/x_{IQR}(n_i)$.

Step 5: The original individual values are removed in each subgroup, when the residuals breach the calculated limits in *Step 4*.

Step 6: After eliminating individual outliers, a new estimate of the standard deviation is obtained from the mean of standard deviations of remaining subgroups for Phase II analysis. Each subgroup standard deviation is adjusted by dividing $c_4(n_i)$.

3.3.3 Estimation of μ for Phase II

Step 1: Obtain the unbiased estimate for standard deviation ($\hat{\sigma_x}$) using the above procedure.

Step 2: For $\bar{x}$, instead of the traditional $\bar{x}_i$ of each subgroup, the trimeans of each subgroup is calculated; $TM_{(m)}$ denotes the m^{th} ordered value of the subgroup trimeans. The trimean of subgroup i is defined by, $TM_i = (Q_{i,1} + 2Q_{i,2} + Q_{i,3})/4$, where $Q_{i,2}$ is the median and $Q_{i,1} = x_{i,(a)}$ and $Q_{i,3} = x_{i,(a)}$ the first and third quartiles with $x_{i,(m)}$ the m^{th} order statistic in subgroup i and $a = \lceil n_i/4 \rceil$, $b = n_i - a + 1$.

Step 3: The 10% trimmed mean: TM_{10}, of sample trimeans is used to get the control limits for the $\bar{x}$ chart as follows:

$$\overline{TM}_{10} = \frac{1}{k - 2\lceil k/10 \rceil} \left[\sum_{m=\lceil k/10 \rceil + 1}^{k - \lceil k/10 \rceil} TM_{(m)} \right]$$

The upper and lower control limits are given by $\overline{TM}_{10} \pm 3\hat{\sigma_d}/\sqrt{n_i}$.

Step 4: Those subgroups whose TM_i fall outside the control limits are removed and the average of the trimeans ($\overline{TM}'$) in the remaining subgroups is used to identify the individual outliers for the remaining unaffected subgroups.

Step 5: Remove individual observations falling outside the limits $\overline{TM}' \pm 3\hat{\sigma_d}$.

Step 6: The mean of remaining subgroups (with remaining individual observations) is used for computing mean ($\bar{\bar{x}}$) and the associated Phase II control limits.

3.4 Shewhart control charts for individual observations

We will be later investigating the autocorrelation in stock returns in Chapters 7 and 8. To retain the autocorrelation, daily stock returns were considered as individual observations to partition the common and special cause periods. The following procedure is from Montgomery (2011, pp.260-261).

The moving range (MR) for the two successive time periods defined as $MR_i = |x_i - x_{i-1}|$ can be used as a basis to identify the common cause variation. Let $\bar{x}$ and $\overline{MR}$ be the average of data series and the moving ranges respectively. The Shewhart control limits are then given by $\bar{x} - 3\frac{\overline{MR}}{d_2}$ and $\bar{x} + 3\frac{\overline{MR}}{d_2}$ where d_2 is bias correction constant.

An observation is then deemed as subject to special causes if the above control limits are breached. Such observations can be removed and the control limits can then be re–calculated using the new $\bar{x}$ and $\overline{MR}$ values. Several iterations may be needed to remove the stock returns subject to special causes. Given that the control limits can also produce false alarms, the following rule for terminating the revision process was applied. The probability of an individual data point falling outside the control limits is 0.0027. For n observations, $0.0027n$ points will be falsely identified points subjected to special causes. If the number of flagged observations is below the Poisson upper control limit of $0.0027n + 3\sqrt{0.0027n}$, the data removal process is stopped.

A robust estimator for the standard deviation (σ) was also considered for a given sample of N individual observations, (see Braun and Park, 2008). The following "Biweight A" (see Lax, 1985, p.739) estimator of σ was preferred:

$$\widehat{S}_c = \frac{N}{(N-1)^{1/2}} \frac{\left(\sum_{|u_i|<1}(x_i - T)^2(1-u_i^2)^4\right)^{1/2}}{\left|\sum_{|u_i|<1}(1-u_i^2)(1-5u_i^2)\right|} \tag{3.3}$$

where T is the sample median, $u_i = \frac{(x_i - T)}{(c\text{MAD})}$ (c is a tuning constant and $c = 7$ was used in this analysis) and MAD is the median absolute deviation from the median: let $d_i = |x_i - T|$

be the absolute deviation of x_i from the median. The MAD is defined by the median of the absolute deviations d_i from the median, MAD = med $\{d_1, d_2, \cdots d_n\}$. A signal rule based on the control limits $\bar{x} \pm 3\hat{S}_c$ was used.

Stock returns associated with special causes were identified iteratively using the same termination rule discussed earlier. Both moving range and the robust estimator performed very similarly in identifying the common and special cause periods. Hence the moving range based estimate is considered as desirable due to its simplicity.

3.5 R package: QCCTS (Quality Control Charts for Time Series)

R codes were written for the procedures described in Sections 3.2, 3.3 and 3.4. Then the QCCTS-R package was developed including newly written functions.

In this study, the package was used to test the validity of the Shewhart's principles for stock return series. There are several types of financial time series data (see Kovářík et al., 2015) and numerous applications in health care, manufacturing, agriculture sectors (see Alwan and Roberts, 1988; Knoth and Schmid, 2004; De Vries and Reneau, 2010; Fretheim and Tomic, 2015) for control charts. The QCCTS-package offers a number of functionalities for control charts of time-indexed data, including;

- Subgrouping for given time intervals such as weekly, fortnightly, monthly and bi-monthly, quarterly (from daily data)
- Calculating the sample moments (mean, standard deviation, skewness, kurtosis) and co-moments (co-variance, co-skewness, co-kurtosis) in common and special cause periods
- Partitioning the common and special cause periods based on Shewhart S chart criterion (standard and robust estimators can be used for σ)

- Shewhart $\bar{X}$ chart and S chart for time series data (for subgroups and individual observations)
- Implementing the robust estimators for σ and μ. (Sections 3.3 and 3.4)

The help documentation of the QCCTS-package is also available in https://github.com/npremara/QCCTS. The QCCTS package passed `R-cmd-check` and will be available in https://cran.r-project.org/ after further improvements.

3.6 Summary

This chapter presented the methodology and software used. The decision rules based on the Shewhart S chart were described for partitioning the total data into common and special cause periods. Both cases of subgrouped data and individual observations are considered for partitioning.

Using empirical data, the partitioning of common/special cause periods, the effect of subgroup size, and behaviour of the first four moments in partitioned data were demonstrated. Robust estimators for σ and the consequent robust charting procedure were used in partitioning in order to minimize the effect of outliers.

All these techniques will be researched further in the coming chapters. The R-package: QCCTS developed exclusively for this study and discussed in this chapter has a wider potential use in the implementation of Shewhart control charts for any (non-financial) time series data.

Chapter 4

Market trade–offs

In this chapter, *Proposition* 1 stated below is examined in relation to the risk–return trade–off issue (identified in section 1.1, *Issue* 1).

Proposition 1: The distribution of stock returns will follow a normal distribution under common cause variation.

To justify *Proposition* 1, the distributional properties of returns in the common and special cause periods are examined. Mean, standard deviation, skewness and kurtosis are chosen as particularly relevant in this context. As exemplars of *Proposition* 1 in action, the non-normality is investigated using these first four moments for log returns of S&P 500 stocks. In order to address the risk–return trade–off issue, the relationships between the first four moments are studied assuming known theoretical distributions for return series. This approach also allows the models to be compared with empirical data. Much of this chapter appeared in the publication Premarathna et al. (2016b).

4.1 Introduction

Mandelbrot (1963), Scott and Horvath (1980) and Peiró (1999) considered the implications of excess skewness and kurtosis in portfolio analysis. Importance of the first four moments in designing portfolios was also illustrated in the work of Simkowitz and Beedles (1978), Mills (1995), Premaratne and Bera (2000) and Pierro and Mosevich (2011). This literature motivated us to investigate the first four moments in explaining the variation of stock returns over the common and special causes identified using Shewhart's framework.

Damodaran (1985) was the first to highlight that negative skewness can result from event structure as well as from the bias (good and bad) in the information structure. Chen et al. (2001) proposed the price discovery (bull market and bear market effect) process as another reason for skewness. Excess kurtosis in financial returns is well known and according to Damodaran (1985) exists as a result of market stability or asymmetric balance between “bull and bear" market activities (i.e. event structure) and the frequency of information releases relative to natural event frequency (events that change the value of share price of a company). The quality of information coming into the market controls a trade–off between skewness and kurtosis (Calvet and Fisher, 2007). It is important therefore to assess the empirical validity of this theory and to investigate the sources of variation which lead to changes in market dynamics.

The fundamental purpose of financial economics is to examine the way the expected return in an investment changes over time, the possibility of explaining excess returns due to changes in volatility and changes in average return per unit risk over time. A trade–off is a technique of exchanging one or more desirable outcomes for increasing or obtaining other worthwhile outcomes in order to maximize the total return or effectiveness under given circumstances. The significance of the mean/standard deviation and skewness/kurtosis trade–offs is therefore examined for each of the two partitions of the total data.

Movements in stock prices (returns) and trading volumes are influenced by new information flowing into the market. Price–volume, returns–volume and volatility–volume relationships have been examined in the literature; Andersen (1996) investigated the trading volume as a factor of the return volatility process; Darrat et al. (2007) examined the volume–volatility relationship for samples partitioned on a basis of with and without identifiable public news for large and small stocks trading in NYSE; Karpoff (1987) showed empirically that average volume is positively correlated with the magnitude of the price changes, and argued that this needed to be investigated for event/no-event time periods. Trading volumes in the partitioned data is included in this study .

The distribution of stock returns was viewed as a mixture of special and common cause variations in Chapters 2 and 3. The technique of rational subgrouping was used to typify variation as either due to common or special cause. The process for separation of rational subgroups into common and special causes was based on excess standard deviation within a specified subgroup of data. This whole procedure was described in Chapter 3 (Section 3.2). Throughout this chapter, weekly subgroups of the daily log returns from adjusted closing price for 428 stocks of S&P 500 companies were used.

This chapter is organized as follows: A detailed description of the variations in financial data from S&P 500 companies; particularly the three-way treatment of total, common and special cause variations of stock returns is provided in Section 4.2. Section 4.3 demonstrates the mean/standard deviation and skewness/kurtosis trade–offs observed for the known probability distribution models. The observed patterns are then used to assess the validity of risk–return trade–offs for real data. In Section 4.4 and 4.5, mean/standard deviation and skewness/kurtosis trade–offs are given for real data and Section 4.6 explores the trading volumes in the partitioned data. Section 4.7 summarises the chapter.

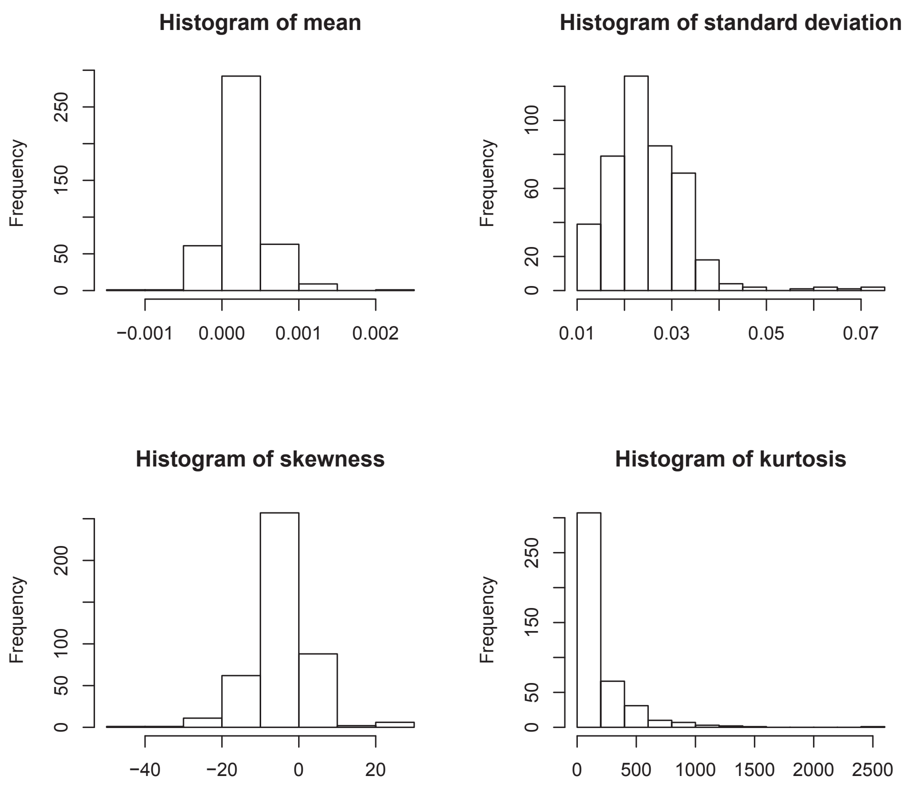
4.2 Descriptive analysis

Returns on each of the 428 S&P 500 companies (more details were given in Section 1.4, an arbitrary ten years time period was chosen from 2003 to 2013) were calculated as logarithmic price changes. Figure 4.1 presents the histograms of summary statistics, including mean, standard deviation, skewness and kurtosis. The combination of the four moments of the stock returns for individual stocks cannot be easily identified by looking at these plots of the data. In Section 3.2, Table 3.2 showed the variations in the four moments total and partitioned stock returns. The normality of the stock return distributions was statistically examined based on the descriptive statistics calculated from the 428 series of returns.

The means of the returns range from -0.0011 to 0.0024 while standard deviations range from 0.0104 to 0.0748. The stock, Level 3 Communications showed the highest positive skewness 25.2584 while Berkshire Hathaway had the highest negative skewness of –49.7960. The kurtosis for all returns distributions was greater than three (5.2938 to 2572.4923) implying excess kurtosis for all series; Berkshire Hathaway gave the most extreme kurtosis. The Jarque-Bera (JB) test statistic (Jarque, 2011) was also used to confirm the non-normality. A minimum value for this test statistic of 610.0956 was found across the 428 stocks, which is very large when compared with a Chi-square distribution with two degrees of freedom; the idea that any of the original series of returns are normally distributed is therefore rejected.

When looking at the mean return for each series, the hypothesis $H_0 : \mu_k = 0$ versus $H_a : \mu_k \neq 0$ is tested, where μ_k denotes the true mean. The t-test statistic $t = \frac{\bar{X}_k}{S_k/\sqrt{N}}$, where $\bar{X}$ and S_k denote the sample mean and standard deviation of each returns series based on all N observations, had p-values greater than 0.05 for 416 of the 428 returns. Thus the null hypothesis of zero mean was not rejected at the 5% level of significance in most of the series.

Figure 4.1: The first four sample moments for stock returns in S&P 500 list



With respect to the skewness of each series Skew_k, the hypothesis tested was H_0 : $\text{Skew}_k = 0$ versus $H_a : \text{Skew}_k \neq 0$. The test statistic is $t = \frac{\widehat{\text{Skew}}_k}{\sqrt{\frac{6}{N}}}$ (Stuart and Ord, 1994) and the sample skewness is:

$$\widehat{\text{Skew}}_k = \frac{\frac{1}{N}\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(X_{ij} - \bar{\bar{X}}_k\right)^3}{\left(\frac{1}{N}\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(X_{ij} - \bar{\bar{X}}_k\right)^2\right)^{3/2}} \tag{4.1}$$

where $\overline{\overline{X}}_k$ is the overall mean return in the kth series of returns. The null hypothesis of zero skewness was rejected for 312 out of the 428 stocks. In other words there was evidence that many but not all companies show excess skewness in their daily returns.

Finally, with respect to considering the excess kurtosis, the hypothesis, $H_0 : \text{Kurt}_k = 3$ versus $H_a : \text{Kurt}_k \neq 3$ was tested using the test statistic $t = \frac{\widehat{\text{Kurt}}_k - 3}{\sqrt{\frac{24}{N}}}$, where sample kurtosis,

$$\widehat{\text{Kurt}}_k = \frac{\frac{1}{N}\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(X_{ij} - \bar{\bar{X}}_k\right)^4}{\left(\frac{1}{N}\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(X_{ij} - \bar{\bar{X}}_k\right)^2\right)^2} \tag{4.2}$$

considered for each series of returns. Thus the p-values were all close to zero, and the null hypothesis of $\text{Kurt}_k = 3$ was rejected for all stocks.

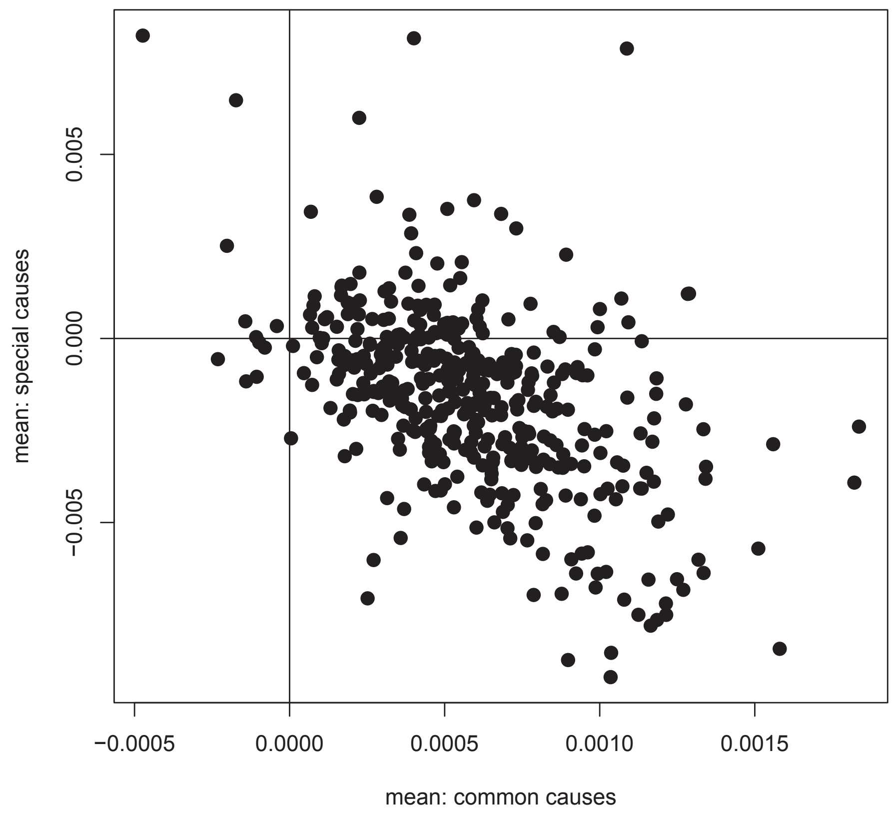
The above analysis for all data for each stock was repeated after the rational subgroups had been separated into partitions based on their display of common or special cause variation. This procedure was explained in Section 3.2. In the common cause subgroups, the mean and standard deviation for stock returns varies from –0.0004 to 0.0018 and 0.0074 to 0.0288 respectively. Excess skewness disappears for all series (–0.3270 to 0.2509) while the kurtosis ranges from 3.2461 to 5.8858. Repeating the hypothesis tests shown above for those returns found within the common cause subgroups gave the following results. The JB test statistics were sharply decreased with a (significant) minimum of 10.4824; the null hypothesis of zero mean return was rejected at the 5% level of significance for 216 out of the 428 series ; The test for zero skewness was rejected at the 5% level for only 96 out of the 428 series of returns; and the excess kurtosis is still significant in all 428 companies .

The results for the subgroups deemed to have fallen outside the control limits for standard deviation are quite informative. The evidence showed us that the high skewness and excessive kurtosis present in the full set of returns was largely driven by the returns in the special cause subgroups. The result suggests that 337 companies showed negative mean returns varying from –0.0091 to 0.0082, in the special cause periods. The skewness of returns in special cause periods ranges from –21.6218 to 12.7836 while the kurtosis varies from 2.7218 to 480.4858. Care needs to be taken when considering the mean return during special cause time periods; the null hypothesis of zero mean was only significant for 6 companies, but the relevance of the mean return was clouded by the large standard deviation of returns during these time periods. Also, the null-hypothesis of zero-skewness rejected for 257 stocks, and excess kurtosis was significant in all series. The descriptive analysis of the empirical data suggested that there was less asymmetry in the returns series during the common cause periods, compared to the total and special cause periods.

As well as considering the evidence from within each of the partitioned sets of subgroups, some relationships induced by the partition are investigated. Among them, particular interest is given to the mean return in common and special cause variations; Figure 4.2 provides the graphical representation of this relationship. The figure suggests a strong negative relationship between the respective variables. Moreover, most of the returns series showed a positive average return in common cause time periods while that in the special cause time periods was negative. The excess volatility might be a result of large falls in stock price series, and the special cause subgroups were identified based on excess volatility. Therefore, the positive and negative mean returns in common and special cause variations respectively are sound evidence for the appropriate partition technique. The inconsistency of the present evidence with initial hypothesis testing due to the high volatility of the special cause variation.

The first four moments observed in the common cause periods are largely consistent with the properties of standard normal distribution. From Table 3.3 in Section 3.2, the excess skewness and kurtosis are mostly induced by unusual changes in the return series. These results, therefore, largely validate the first proposition, namely that the distribution of stock returns follows a normal distribution under common cause variation.

Figure 4.2: Average returns in common and special cause periods



4.3 Trade–offs between the first four moments

In this section, the mean/standard deviation and skewness/kurtosis trade–offs are explored using randomly generated data from symmetric and asymmetric distributions. Random data from the known distributions are considered because these have been used to model stock returns in the literature. Understanding the trade–offs for known distributions is important to validly apply the observed patterns to real data. Data from known distributions are generated so that random sample variations are allowed as this situation is equivalent to having solely common cause data.

In particular, samples were drawn from standard normal, Log-normal (positive skewed), Beta (negative skewed), Student's t , Cauchy, and Weibull distributions. Figure 4.3 gives the mean/standard deviation trade–offs in each distribution. The trade–off from the standard normal distribution is not significant and both negative and positive-skewed distributions show the same sign of significant correlations between the respective moments. The first column in Table 4.1 provides the correlation coefficient between the mean and standard deviation in respective distributions.

Table 4.1: Correlations between mean/standard deviation and skewness/kurtosis for known theoretical distributions

Distribution	Mean/standard deviation	skewness/kurtosis
Normal(0,1)	0.010	−0.008
Student's t(2)	0.121***	−0.004
Beta(10,2)	−0.538***	−0.455***
Log-Normal (0,1)	0.892***	0.765***
Cauchy(0,1)	−0.110***	0.002
Weibull(1,5)	−0.162***	−0.109***

*** - P value ≤ 0.001 or very highly significant.

The stock market is more volatile during periods of unusual events, which also implies inefficiency of the information flow. As volatility increases, investors may observe single or multiple signals beyond their existing beliefs; they quickly change investment decisions. Nevertheless, when volatility decreases or good news is coming into the market, investors react slowly. This asymmetry demonstrates how skewness increases and kurtosis falls as information becomes inefficient. In addition, it is known that information quality differs from stock to stock because there are different changes in volatility.

MacGillivray and Balanda (1988) justified the interrelatedness of skewness, anti-skewness and kurtosis, describing them collectively as the shape of the distribution. Figure 4.4 shows the skewness/kurtosis structures and second column of the Table 4.1 presents the correlation coefficients between the two moments in each of the standard distributions. Each figure shows the inverted bow-tie effect, but the data points highlight the different patterns for the symmetric and asymmetric theoretical distributions. This evidence suggests that the trade–off structures depend on the distributional properties of the underlying distributions.

Figure 4.3: Mean/standard deviation trade–off: known distributions

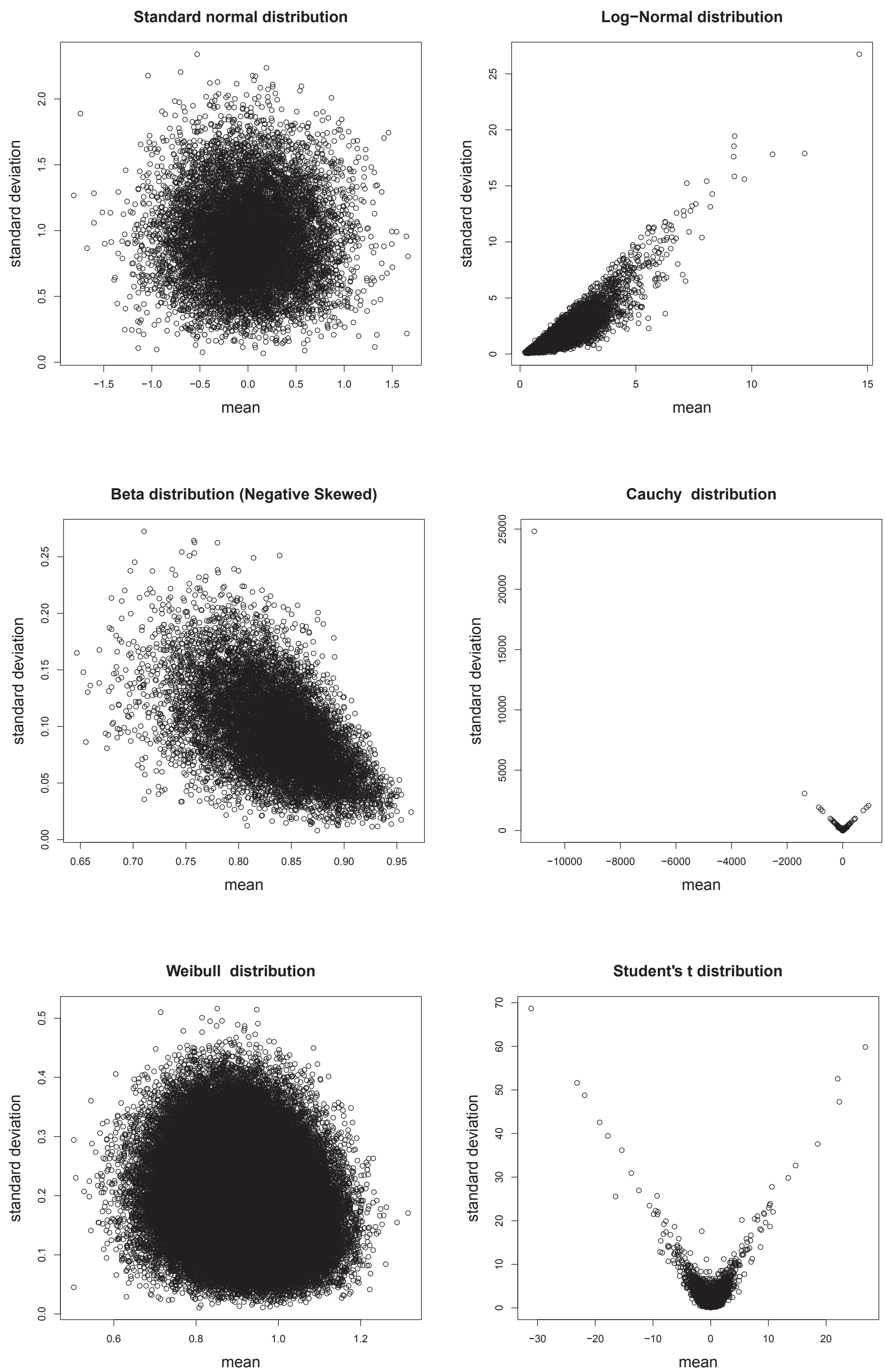
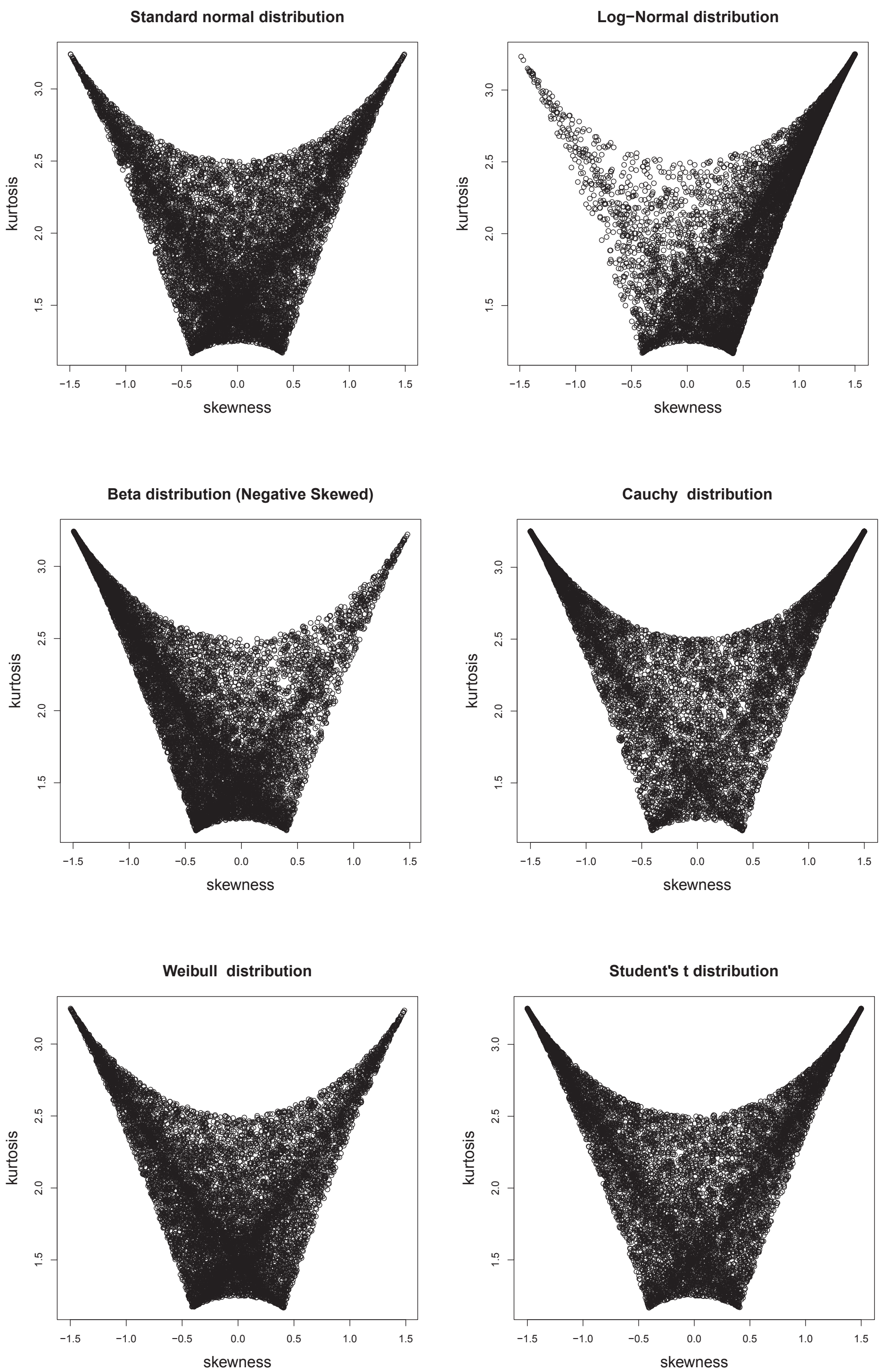


Figure 4.4: Skewness/kurtosis trade–off: known distributions



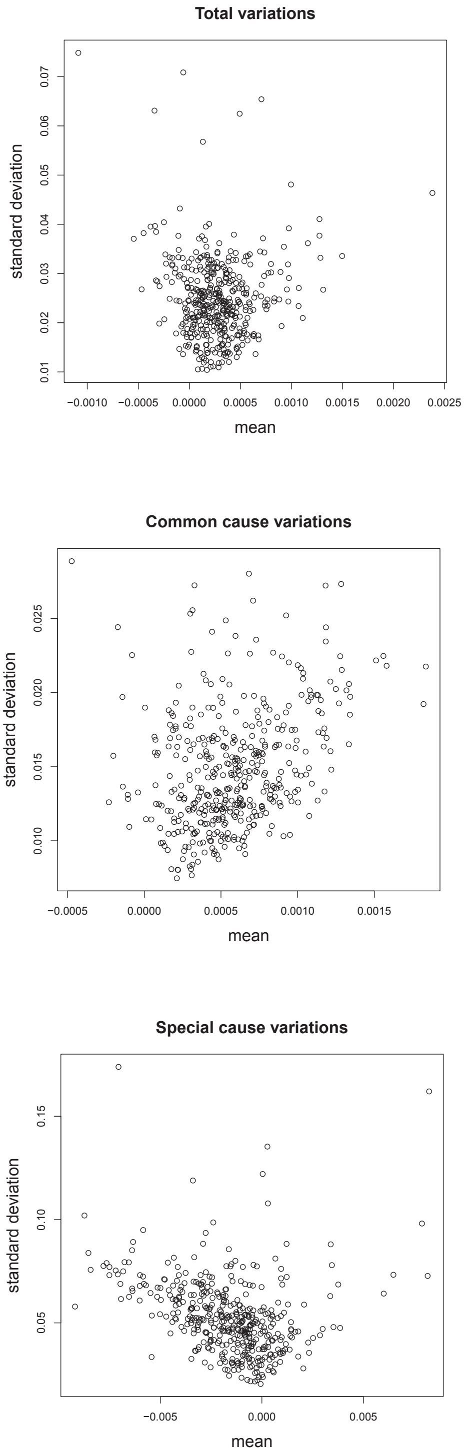
4.4 The mean/standard deviation trade–off

A negative mean/standard deviation trade–off is found in special cause periods which is highly significant (correlation: –0.4064), while in the common periods there is a significant positive trade–off (correlation: 0.3280). The partition process, therefore, highlight the significant trade–offs in the common and special cause periods that are not observed in the total data. Figure 4.5 gives a graphical view of the trade–offs in the total, special and common cause time periods.

These results largely justified the mean/standard deviation trade–off observed in the theoretical distributions and the trade–off in common and special cause variations are important for understanding the empirical behaviour of returns series. The partitioning of stock returns using quality control theory has provided a further perspective to the well-understood risk–return trade–off. The positive trade–off found in the common cause periods is consistent with the CAPM theory. Partitioned data demonstrates that a positive risk–return relationship occurs in the absence of return anomalies.

The conclusions based on the empirical data may not be consistent over different time intervals, therefore, the analysis was repeated with varying time intervals of data by considering historical data from 15, 10 and 5 years. Mean/standard deviation trade–off for total and special cause periods are significant and negative in each situation; in common cause periods it is slightly changed from positive (significant) to negative (not significant).

Figure 4.5: Mean/standard deviation trade–off, stocks in the S&P 500 list



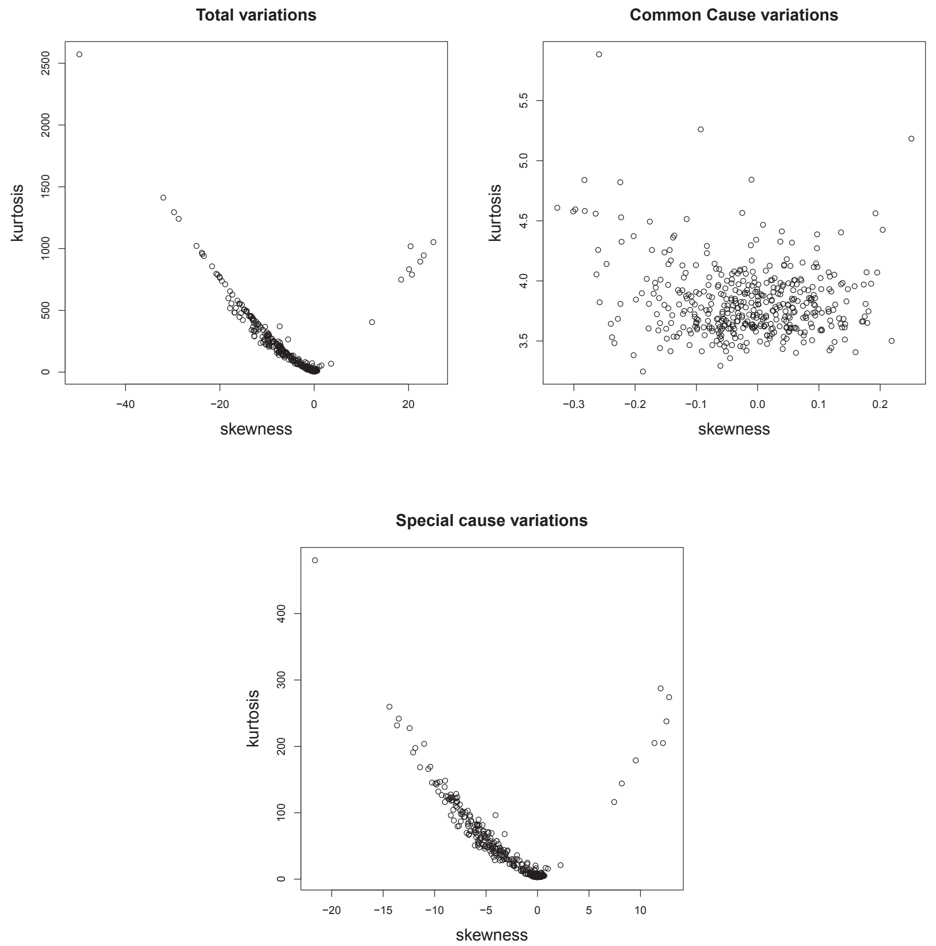
4.5 The skewness/kurtosis trade–off

In this section, the impact of volatility on the inter-relationship between skewness and kurtosis is studied. It is believed that the skewness/kurtosis trade–off is not sufficiently documented elsewhere. For example, several seminal works on modelling approaches for stock returns or market volatility have not considered the skewness/kurtosis trade–off even though they have obtained these summary statistics; see for example, the summary statistics from Table 1 in Ding and Granger (1996, p190).

In this analysis, a highly significant negative skewness/kurtosis trade–off is observed in both total (correlation: –0.7887) and special cause (correlation: –0.7816) periods. Perhaps most important however is that there is no significant relationship (correlation: –0.0316) between skewness and kurtosis in the returns during common cause periods (as seen in Figure 4.6) so it is concluded that the trade–off found over all time periods is largely driven by events during the special cause periods. A negative skewness/kurtosis trade–off is observed for negatively skewed distributions but not from the standard normal distribution in Section 4.3.

The skewness/kurtosis trade–off is a little clouded by the inclusion of the nine stocks in the upper right hand corner which have high positive skewness (sample skewness is greater than 2). Ignoring the results for them and concentrating on the remaining 419 stocks, a very strong trade–off between skewness and kurtosis is found and that should be considered when building a portfolio. Specifically, there are highly negative trade–offs in both total (correlation: –0.937) and special cause (correlation: –0.930) periods, but in the common cause periods the positive trade–off (correlation: 0.0068) is not significant. On the basis of empirical results, a highly significant skewness/kurtosis trade–off is found in total as well as in special cause periods which has not been observed before. Consistent results were found for the skewness/kurtosis trade–off in different time intervals. Many studies were found that discuss the effect of skewness and kurtosis in asset pricing mod-

Figure 4.6: Skewness/kurtosis trade–off, stocks in the S&P 500 list



els(e.g. Rubinstein (1973); Kraus and Litzenberger (1976); Jondeau and Rockinger (2006); Harvey et al. (2010)). The results found from this study suggest that the skewness/kurtosis trade–off is important in understanding information dissemination over the market and the partition procedure based on the Shewhart approach gives additional information to the investors.

In addition to mean/standard deviation and skewness/kurtosis trade–offs patterns, the other correlations between the four moments: mean, standard deviation, skewness and kurtosis in the partitioned data are shown in Table 4.2 for total, common and special cause periods. These relationships may give more information to the investors, for example

Boudt et al. (2015) noted that “most investors would be willing to sacrifice expected return and/or accept a higher volatility in exchange for a higher skewness and lower kurtosis leading to a lower downside risk". For this scenario, mean–skewness and standard deviation–kurtosis correlations are important and from Table 4.2 both correlations are significant in total and special cause periods but not in the common cause periods. This evidence shows that the investigation of the relationships between the moments provides useful information to the investors.

Table 4.2: Correlations between the first four moments for other scenarios in total and partitioned data

	Total	
	skewness	**kurtosis**
mean	0.332^{***}	-0.304^{***}
standard deviation	-0.203^{***}	0.365^{***}
	Common causes	
	skewness	**kurtosis**
mean	0.087	-0.136^{***}
standard deviation	0.205^{***}	0.017
	Special causes	
	skewness	**kurtosis**
mean	0.615^{***}	-0.471^{***}
standard deviation	-0.420^{***}	0.551^{***}

*** - P value ≤ 0.001 or very highly significant.

4.6 Analysis of trading volumes

This section investigates the average volumes, volatility–volume, return–volume and price–volume relationships in the partitioned data. Weekly averages volumes are obtained from the list of stocks given in Section 4.2. The time index in common and special cause periods is used to retrieve the average volumes and prices.

The t-statistic of the null-hypothesis: “log ratio of average trading volumes in special cause periods to the common cause period is equal to zero" is tested and the t-statistic value is 47.221 which is highly significant. So, it appears that in the periods with special causes, the trading volume is significantly different from that in common cause periods perhaps due to the information asymmetry in the stock market.

The analysis is extended by considering the correlation between weekly volatility and average volume over three partitions. In total data and the common cause periods, a majority (more than 95%) of stocks show a positive significant correlation between volatility and volume but only half of the stocks show a significant relationship in special cause periods. These results are consistent with the empirical findings in Darrat et al. (2007): "overconfident investors overrate the precision of their private news signals and therefore trade too aggressively in the absence of public news; when public news arrives, investors' biased self-attribution triggers excessive return volatility".

A significant difference is not observed in the return–volume relationship across the partitions. The price–volume relationship was investigated in partitioned data: 80% of stocks showed a negative significant correlation between price and volume in total as well as common cause periods; only 60% of stocks showed a significant relationship in special cause periods. These results justify that the trading volumes in the partitioned data also contain useful additional information which must also be considered in investment decision making.

4.7 Summary

The first proposition stated in this thesis has been justified using the first four moments in common and special cause classified real data. Based on this empirical analysis, the following properties of both common and special cause variations were observed.

- The returns in special cause periods were mostly negative, and as a consequence they tend to show both negative mean and negative skewness.

- Positive and negative mean/standard deviation trade–offs were present in the common and special cause periods respectively.
- Given the finding that the skewness/kurtosis trade–offs were found to be largely due to events during special cause time periods, there is a need to alter the way investors design their portfolios.
- Trading volumes in special cause periods are significantly different from those pertaining common cause periods.
- The volatility–trading volumes relationship was found to be significant in total and common cause periods for the majority of stocks but not in special cause periods.

It is established that there is a need to incorporate the trade–offs in financial modelling. In addition, the differing volatility from company to company is shown to have an impact on the skewness-kurtosis trade–off. These findings help to understand the risk–return trade–off through the skewness-kurtosis trade-off. The approach proposed in this thesis shows that the trade–offs found in stock returns, are mainly due to excess volatility during the special cause periods. This chapter concludes that the partitioning of stock returns using Shewhart methodology can be extended to the typify the expected nature of the first four moments for portfolio management.

Chapter 5

Difference control charts

The *Issue* 2 is to address the co-moments (co-variance, co-skewness and co-kurtosis) and the *Proposition* 2 is to assess co-moments in partitioned data. The two variables stock and market returns, are correlated paired variables and hence the partition of common and special causes is desired to be based on the difference series of returns instead of an individual series (without reference to another series). Also, market participants often observe changing patterns in stock returns relative to another asset or index. The difference series between stock and market (or another asset) returns which retain the joint behaviour and one-to-one time oriented relationship is therefore considered.

Published research on control chart procedures for monitoring paired variables or the *difference series* are sparse in the literature. Following an analysis of the average run length properties of the $\bar{d}$ chart, which monitors the mean of the differences between paired variables, a new chart based on S_d, the subgroup standard deviations of the differences, is proposed in this chapter. Much of this chapter appeared in the publication Premarathna et al. (2017).

5.1 Introduction

Numerous control statistics are employed for multivariate process monitoring; for a review, see Bersimis et al. (2007) and references therein. Bivariate process monitoring has received particular attention in the literature; see Grabov and Ingman (1996); Khoo (2004); Riaz and Does (2008); Abu-Shawiesh et al. (2014). This chapter focuses on bivariate control chart procedures for paired variables. The natural correlation between the paired variables enables their difference to be used as a powerful measure for making comparisons.

Grubbs (1946) proposed a control chart procedure based on the difference of two independent sample means, such as the current production sample and the reference lot sample; see Chapter 9 of Montgomery (2011) for a discussion of this chart. On the other hand, Ott (1947) proposed another chart based on the mean of the differences for paired variables which are expected to be correlated. The difference chart of Ott (1947) is useful for calibration studies and elaborated in Chapter 4 of Ryan (2011). Tracy et al. (1995) considered using the correlation coefficient for monitoring paired data arising from a bivariate normal distribution, and created a control statistic for the existing chi-square chart or Hotelling's T^2 chart.

The chart based on the mean of the paired differences is designated as the $\bar{d}$ chart. The well known S chart methodology is further employed for the subgroup standard deviations of the paired differences and designate the resulting chart as the S_d chart. Even though the $\bar{d}$ chart was proposed in the middle of last century, its average run length (ARL) properties were not investigated. The run length properties of $\bar{d}$ and S_d charts are studied here, assuming a bivariate normal distribution for the paired variables. Many out-of-control scenarios can be considered for bivariate data allowing for shifts in the elements of both mean vector and covariance matrix; a selection of these are addressed in this chapter. The ARLs of the $\bar{d}$ chart are compared with chi-square chart analogue of the Hotelling's T^2 chart (used when standards are unknown) and individual $\bar{X}$ charts. The

ARL performance of the S_d chart is compared with the $|S|$ control charts after allowing for shifts in the covariance matrix. Run length properties of these charts are compared for known (fixed) out-of-control situations when standards are known.

In practice, these charts are used when limited retrospective data is available. Many authors, including Jensen et al. (2006); Chakraborti (2007); Braun and Park (2008); Faraz et al. (2015) have examined the effect of estimated parameters on control limits for various control charts. The in-control and out-of-control performance of the difference charts are investigated when parameters are therefore estimated from limited Phase I data.

In order to investigate both aleatory and epistemic uncertainties on control chart performance, run lengths are compared using an inverse-Wishart prior on the covariance matrix. This allows an assessment of which charts are superior for monitoring stock returns or other scenarios where a high level of contamination exists over the long run.

The rest of this chapter is organized as follows: Section 5.2 describes the $\bar{d}$ and S_d chart procedures and discuss the determination of their control limits. Section 5.3 gives a summary of our simulation study for comparing the run length performance of the difference charts. In Section 5.4, a finance application is presented; historical daily data from Apple Inc. and the S&P 500 index are used for illustration. The chapter is summarised in Section 5.5.

5.2 Design of $\bar{d}$ and S_d charts

Let (X, Y) be two correlated variables having a common pairing variable, and $d = X - Y$ be their difference. The probability distribution of d is normal when the joint density of X and Y is bivariate normal; see pp.67-75 in Springer (1979) for the proof. Let us assume that k subgroups of size n each are formed and then their subgroup means $\bar{d}$'s, and standard deviations S_d's are computed. Consider, the data series $\left(X_{ij}, Y_{ij}\right)$ where

$i = 1, 2, ..., k \quad j = 1, 2, ..., n$. Let, $d_{ij} = X_{ij} - Y_{ij}$, then,

$$\bar{d}_i = \frac{1}{n}\sum_{j=1}^{n} d_{ij}, \qquad S^2_{d_i} = \frac{1}{n-1}\sum_{j=1}^{n}\left(d_{ij} - \bar{d}_i\right)^2$$

The control limits for the $\bar{d}$ and S_d charts will then be the same as the limits set for the traditional (Shewhart) $\bar{X}$ and S charts. For the case of given parameters (or standards known case) μ_X, μ_Y, σ^2_X, σ^2_Y and ρ, the three-sigma upper control limit (UCL) and lower control limit (LCL) for the $\bar{d}$ chart are $\mu_d \pm 3\frac{\sigma_d}{\sqrt{n}}$ where $\mu_d = \mu_x - \mu_y$ and $\sigma^2_d = \sigma^2_X + \sigma^2_Y - 2\rho\sigma_X\sigma_Y$. For the matching probability limits for S_d chart are,

$$UCL = \sqrt{\frac{\chi^2_{n-1;0.99865}}{n-1}}\sigma_d$$
$$LCL = \sqrt{\frac{\chi^2_{n-1;0.00135}}{n-1}}\sigma_d$$

when the false alarm rate $\alpha = 0.0027$ is equally split between the lower and upper sides. The performance of the control charts are discussed using control limits based on three-sigma; users should choose a false alarm rate that suits their scenario, and adjust the control limits accordingly.

For the parameters unknown case, the sample estimates for μ_d and σ_d are used from the difference series. The control limits for the $\bar{d}$ chart are $\bar{\bar{d}} \pm 3\frac{\bar{S}_d}{c_4(n)\sqrt{n}}$. For the S_d chart, the probability limits become $\frac{\bar{S}_d}{c_4(n)}\sqrt{\frac{\chi^2_{0.99865}}{n-1}}$ and $\frac{\bar{S}_d}{c_4(n)}\sqrt{\frac{\chi^2_{0.00135}}{n-1}}$ where $\bar{\bar{d}}$ is the average subgroup mean of the differences and $\bar{S}_d$ is the average subgroup standard deviation of the differences. When standards are unknown, control limit constants such as c_4 can be used for the $\bar{d}$ and S_d charts. Tabulated values for c_4 etc. are given in many textbooks, e.g. Montgomery (2011).

5.3 Run length properties of the difference charts

A control chart's ARL, being the expected number of plotted points before a signal, measures the power of the charting procedure as a function of the process parameters. Two control charts having the same in-control ARL can be compared for their performance to detect an out-of-control situation. Control charts are expected to achieve minimum ARLs for a variety of out-of-control situations. The ARL performance of the $\bar{d}$ chart is compared with the chi-square control chart for its ability to detect shifts in the means (μ_X and μ_Y) when standards are known. The Hotelling's T^2 chart was replaced with the chi-square control chart when parameters are estimated from Phase I data. The ARLs of the S_d chart were compared with the $|S|$ chart. The W chart of Alt (2004) was also examined in addition to the $|S|$ chart. The ARL performance of the W chart was rather poor and hence a detail discussion is not given. Table 5.1 gives the control statistic and control limits for each chart that have been used in the comparative analysis.

If μ_d and σ_d are known (implying that $\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2$ and ρ are given), then one could calculate the ARLs in a straight forward way because the distribution of run length is geometric. For the case where μ_d and σ_d are estimated (the standards unknown case), derivation of explicit formulae for ARLs is mathematically hard. The geometric distribution based ARL computation will not apply in this situation because the probability of a false signal depends on the estimated control limits (Jensen et al., 2006; Chakraborti, 2007). As a consequence, Monte Carlo simulations are implemented using R statistical software (R Core Team, 2015) to accommodate both situations.

Out-of-control signals are due to changes in one or more of the parameters (μ_X, μ_Y, σ_X^2, σ_Y^2 and ρ) in the bivariate normal model. A selection of changes in means and variances/-covariance is considered. The calculation of RLs for a given type of shift involves the steps given in Algorithm 5.1.

Table 5.1: Control statistics and control limits used for performance comparison when standards are known and estimated

Statistic	Lower control limit (LCL)	Upper control limit (UCL)
$\chi_0^2 = n\left(\bar{X}-\mu\right)'\Sigma^{-1}\left(\bar{X}-\mu\right)$ Montgomery (2011), p.501	–	$\chi^2_{\alpha,2}$
$T^2 = n\left(\bar{X}-\bar{\bar{X}}\right)'S^{-1}\left(\bar{X}-\bar{\bar{X}}\right)$ Montgomery (2011), p.503	–	Phase I: $\frac{p(m-1)(n-1)}{mn-m-p+1}F_{\alpha,2,mn-m-1}$
	–	Phase II: $\frac{p(m+1)(n-1)}{mn-m-p+1}F_{\alpha,2,mn-m-1}$
$\lvert S\rvert$ Alt (2004), p.9	$\lvert\Sigma\rvert\frac{\left(\chi^2_{2n-4,1-\left(\frac{\alpha}{2}\right)}\right)^2}{4(n-1)^2}$	$\lvert\Sigma\rvert\frac{\left(\chi^2_{2n-4,\left(\frac{\alpha}{2}\right)}\right)^2}{4(n-1)^2}$
$\lvert S\rvert$ Alt (2004), pp.9-11	$\frac{\lvert S\rvert}{b_1}\frac{\left(\chi^2_{2n-4,1-\left(\frac{\alpha}{2}\right)}\right)^2}{4(n-1)^2}$	$\frac{\lvert S\rvert}{b_1}\frac{\left(\chi^2_{2n-4,\left(\frac{\alpha}{2}\right)}\right)^2}{4(n-1)^2}$
	where $b_1 = (n-1)^{-2}\prod_{i=1}^{2}(n-i)$	

Algorithm 5.1: Steps for calculating RLs

Step 1: Set run length: $RL = 0$.

Step 2: Generate a random subgroup of bivariate data size m for a given shift in one or two of the parameters, $\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2$ and ρ.

Step 3: Calculate the chosen control statistic (CS) depending on the type of the chart.

Step 4: If $LCL < CS < UCL$, set $RL = RL + 1$ and go to *Step* 2; else terminate and return $RL - 1$. Here LCL and UCL are the in-control lower and upper control limits of the chosen control statistic.

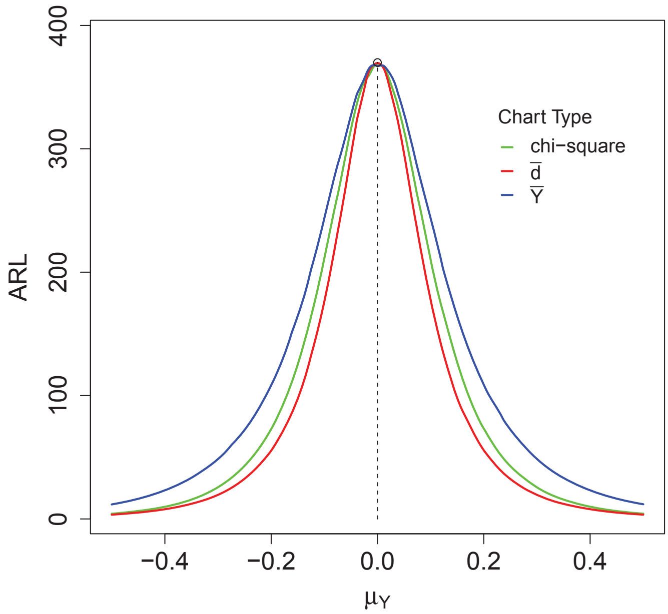
Step 5: Repeat *Steps* 1 to 4 for a large number simulation runs and obtain the large scale Monte Carlo estimates of ARLs using the simulated RL series. Note that $ARL = \mathrm{E}[RL]$, where $RL = min\{i, \quad CS_i \leq LCL \quad \text{or} \quad CS_i \geq UCL, \quad i = 1, 2, 3, ...\}$.

5.3.1 Discussion of out-of-control situations when standards are known

To investigate the run length properties, various out-of-control situations were considered by changing one or two of the five parameters ($\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2$ and ρ). The fixed level shifts are made in the mean vector (**Cases 1-3**) and covariance matrix (**Cases 4-7**) to generate subgroups of random bivariate data, while ensuring that the covariance matrix was positive definite using `mvrnorm` function in R. The parameters $\mu_X = 0, \mu_Y = 0, \sigma_X^2 = 1, \sigma_Y^2 = 1$ and $\rho = 0.75$ are chosen for in-control case and hence, $\mu_d = 0$ and $\sigma_d^2 = 0.5$.

The ARL curves shown in the presented figures employed the `smooth.spline` R function in order to remove the unavoidable minor wiggly patterns caused by the Monte Carlo simulation technique. A number of out-of-control situations are described below and are followed by comments on the performance of the difference charts for a subgroup size

Figure 5.1: (a) ARLs for shifts in μ_Y ($\mu_X = 0$, $\sigma_X^2 = 1, \sigma_Y^2 = 1$ and $\rho = 0.75$)

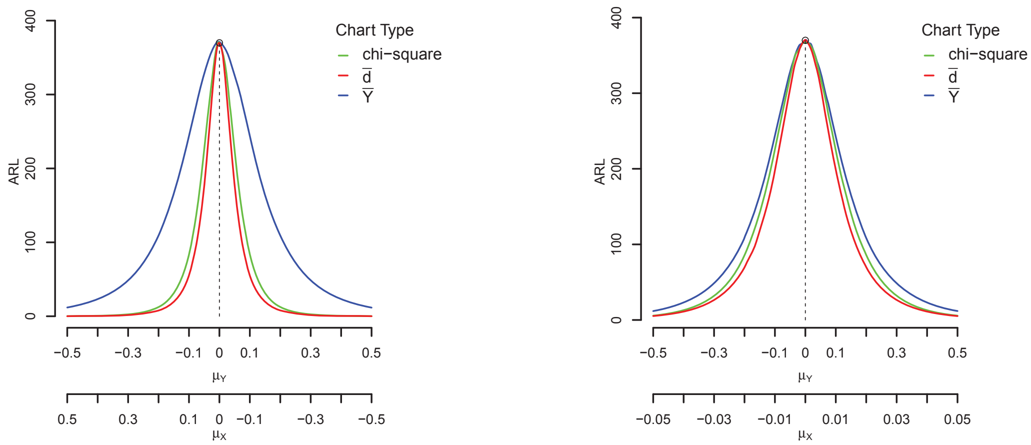


of ten ($n = 10$); similar results were obtained for other subgroup sizes ($n > 10$). To obtain in-control ARL of 370 ($\alpha = 0.0027$) 5000 replicates are used and 2000 replicates for each out-of-control RL simulation.

Case 1: Shifts in one mean

A shift in μ_Y is first considered with fixed μ_X, σ_X^2,σ_Y^2 and ρ. Figure 5.1 shows the ARLs for the three charts when the shift in μ_Y is at a fixed level. The $\bar{d}$ chart showed a faster response in detecting the shifts in the mean than the chi-square chart and $\bar{Y}$ chart in all examined cases. When the shifts are considerably large, all these charts perform equally well.

Figure 5.2: ARLs for (a) equal shifts in μ_X and μ_Y in the opposite direction and (b) unequal shifts in μ_X and μ_Y in the same direction ($\sigma_X^2 = 1, \sigma_Y^2 = 1$ and $\rho = 0.75$)



(a) **(b)**

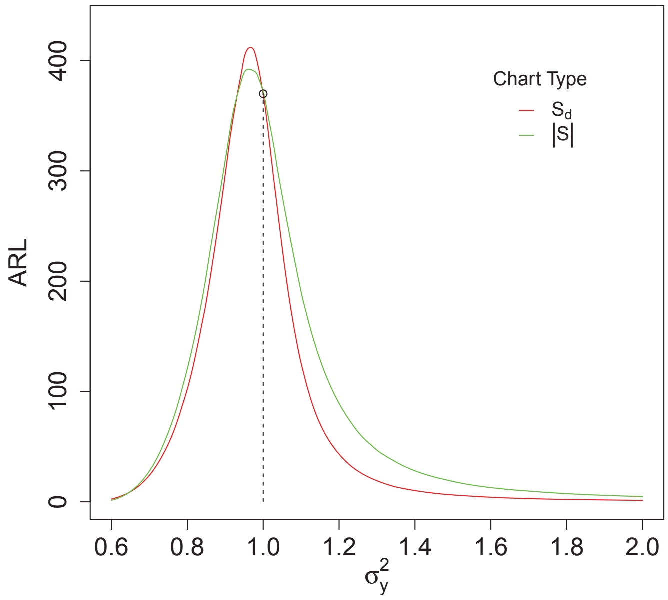
Case 2: Equal shifts in both means

Upward and downward shifts of equal size were made in μ_X and μ_Y in opposite directions. Figure 5.2a shows the ARLs for this setting. The $\bar{d}$ chart gives the shortest ARLs compared to the other charts.

Case 3: Unequal shifts in both means

Unequal amounts of shift were made in both μ_X and μ_Y so that μ_X shifts in the same direction, but slower than μ_Y. The ARLs shown in Figure 5.2b confirm that the $\bar{d}$ chart performs slightly better than the chi-square chart and $\bar{Y}$ chart in detecting these shifts.

Figure 5.3: ARLs for shifts in σ_Y^2 ($\mu_X = 0, \mu_Y = 0, \sigma_X^2 = 1$ and $\rho = 0.75$)



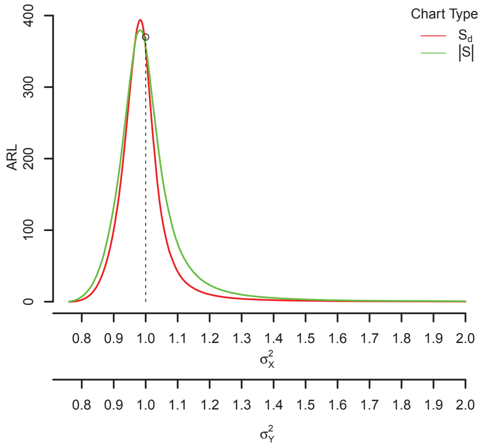
Case 4: Shifts in individual variance

Figure 5.3 presents the ARLs for fixed shifts in σ_Y^2. The performance of the S_d chart is better for detecting upward shifts. For downward shifts in σ_Y^2, both S_d and $|S|$ charts show nearly equal ARLs.

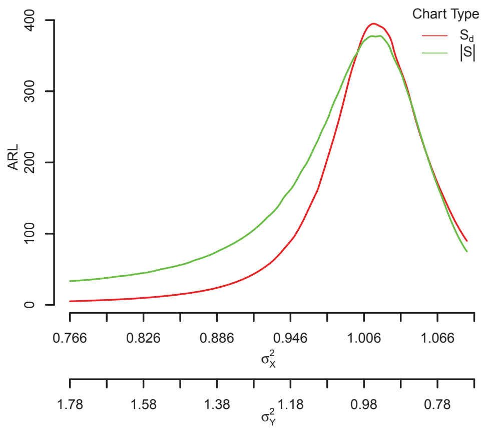
Case 5: Equal shifts in both variances

Figure 5.4a shows the ARLs when both the variances σ_X^2 and σ_Y^2 increase or decrease in the same direction. Even though the charts are affected by the variance shifts, both the S_d chart and $|S|$ charts respond quickly to variance shifts. The S_d chart performs only slightly better than the $|S|$ chart.

Figure 5.4: ARLs for (a) shifts in σ_X^2 and σ_Y^2 in the same direction and (b) unequal shifts in σ_X^2 and σ_Y^2 in opposite directions ($\mu_X = 0, \mu_Y = 0$ and $\rho = 0.75$)



(a)



(b)

Case 6: Unequal shifts in both variances

Unequal shifts in σ_X^2 and σ_Y^2 are considered in opposite directions. Figure 5.4b shows that the S_d chart performs better than the $|S|$ chart for detecting unequal shifts in variances.

Case 7: Shift in correlation coefficient

A shift in $\rho = 0.75$ is made to a value in the range -0.9 to 0.9. Figure 5.5 gives the ARLs values for shifts in ρ while all other parameters are kept constant. The $|S|$ chart shows bimodality in the ARLs. This means that these charts are insensitive to the sign of the correlation coefficient. This is not surprising because the $|S|$ chart involves the determinant of the covariance matrix which can remain the same irrespective of the sign of the correlation coefficient when the diagonal elements remain constant. Ryan (2011, p.328), noted that this can make the $|S|$ chart misleading. From these results, it is believed that the S_d

Figure 5.5: ARLs for shifts in ρ ($\mu_X = 0, \mu_Y = 0, \sigma_X^2 = 1$ and $\sigma_Y^2 = 1$)

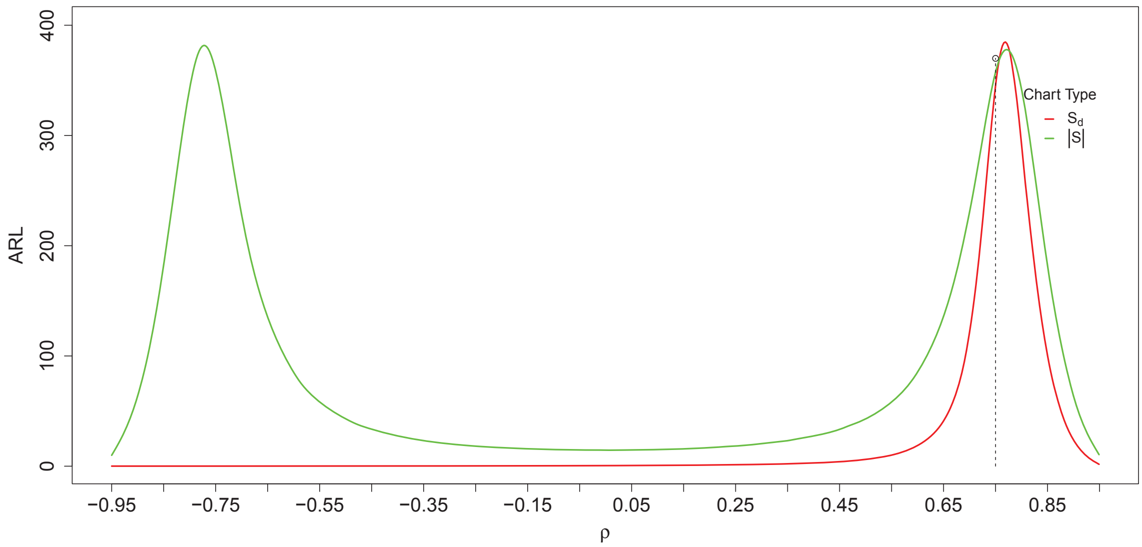


chart addresses this issue and therefore performs better in detecting shifts in ρ. As a consequence, the ARL performance of the S_d chart does not show bimodality.

For the seven out-of-control situations we studied in the subsection 5.3.1 when parameters are known, both $\bar{d}$ and S_d charts performed well compared to their counterparts. However the $\bar{d}$ chart does not detect equal size shifts in both means in the same direction very well. The S_d chart was found not to be sensitive to equal shifts in variances in the opposite direction.

The overall performance of the difference charts is found to be better in general. The $\bar{d}$ chart is a stand-alone chart because we can identify the individual shifts in the mean from it. When compared to the chi-square and $|S|$ charts, $\bar{d}$ and S_d charts are simpler to implement. The control statistic $\bar{d}$ can more readily be given a physical meaning compared to the chi-square and $|S|$ statistics. The traditional $\bar{X}$ chart is robust because non-normality does not affect its performance very much; see Schilling and Nelson (1979). On the other hand, S charts are sensitive to non-normality. Multivariate control charts are known not to be robust to non-normality. For the bivariate case, the $\bar{d}$ chart is expected to be less sensitive to non-normality because it is similar to the $\bar{X}$ chart. While the S_d chart is not expected to be robust, we do expect it to perform better than the $|S|$ chart.

5.3.2 Discussion of RLs properties when standards are unknown

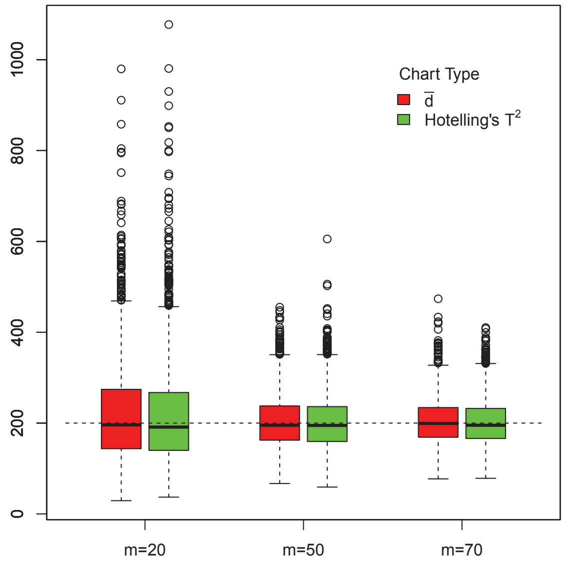
When process parameters are unknown, control limits were calculated from estimated parameters. For uncontaminated data, estimation based on large sample sizes is equivalent to the known parameter case. The case where limited Phase I data is therefore interested. A pairwise comparison was carried out for $\bar{d}$ - Hotelling's T^2 charts and S_d - $|S|$ charts. In-control RL distributions were obtained for the four charts when the number of subgroups $m = 20, 50, 70$, subgroup size $n = 10$ and $\alpha = 0.005$.

In-control ARL was calculated from the run length distribution of 5000 runs. The control limits are now based on the estimated parameters from a bivariate normal sample of size mn. Algorithm 5.1 is followed to obtained the RLs. The expected in-control ARL value is 200 for $\alpha = 0.005$.

Figure 5.6 gives the box-plots for the in-control ARLs of $\bar{d}$ chart and Hotelling's T^2 chart when the parameters are estimated from m subgroups of size $n = 10$. For every chart, 2000 in-control ARLs are used. When $m = 20$, both charts show a large range including a number of outliers. The variability in in-control ARLs was narrow for large subgroup sizes ($m = 50, 70$) and there was no significant difference between the two charts. Based on the figures, it is noticed that the average of in-control ARLs of $\bar{d}$ chart was closer to the expected in-control ARL than the Hotelling's T^2 chart but the standard deviations of in-control ARLs were clouded by the outliers.

The distribution of in-control ARLs for S_d chart was quite different from $|S|$ chart (see Figure 5.7). The interquartile range for in-control ARLs of S_d chart was large compared to $|S|$ chart when $m = 20$ and becomes narrow when the number of subgroups increases.

Figure 5.6: Distributions of in-control ARLs for $\bar{d}$ chart and Hotelling's T^2 chart when the parameters were estimated from m subgroups of size $n = 10$

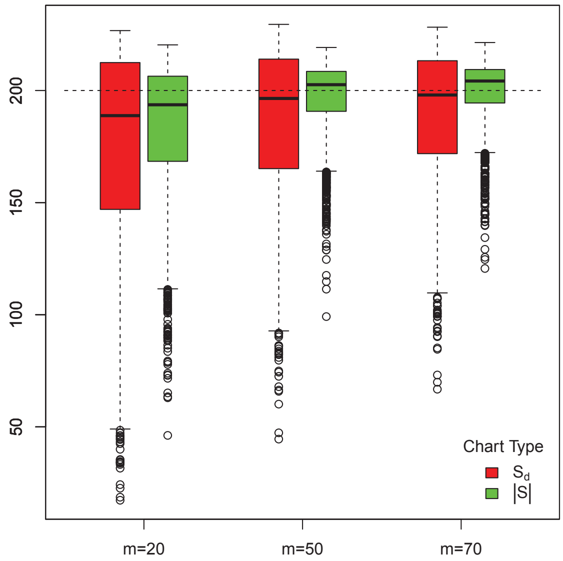


The median in-control ARL of the S_d chart converged to the target value of 200, unlike the $|S|$ chart.

The pairwise chart comparison procedure is employed for in-control ARLs when the parameters are estimated from small samples given by Zwetsloot and Woodall (2017, p.10). This comparison allows evaluation of chart performance as equal, better and eventually worse as compared to the in-control ARL. The definition from Zwetsloot and Woodall (2017) is re-stated using our notation. Let us consider the set of unknown parameters for difference charts and their alternatives as $\Phi = \{\hat{\mu}_x, \hat{\mu}_Y, \hat{\sigma}^2_X, \hat{\sigma}^2_Y, \hat{\rho}\}$, $ARL_{C_1}(\Phi)$ be the in-control ARLs of control chart C_1 and $ARL_{C_2}(\Phi)$ be the in-control ARLs of control chart C_2. Then:

> the performance of chart C_1 is equivalent to C_2 iff
>
> $|ARL_{C_1}(\Phi) - ARL_{C_2}(\Phi)| \leq 0.05 ARL_0$, where ARL_0 is the expected in-control ARL for a given false alarm rate.

Figure 5.7: Distributions of in-control ARLs for S_d chart and $|S|$ chart when the parameters were estimated from m subgroups of size $n = 10$



the performance of chart C_1 is better than C_2 iff

- $ARL_{C_1}(\Phi) < ARL_{C_2}(\Phi)$ when $ARL_{C_1}(\Phi) > ARL_0$ and $ARL_{C_2}(\Phi) > ARL_0$
- $ARL_{C_1}(\Phi) > ARL_{C_2}(\Phi)$ when $ARL_{C_1}(\Phi) < ARL_0$ and $ARL_{C_2}(\Phi) < ARL_0$
- $ARL_{C_1}(\Phi) > ARL_0$ and $ARL_{C_2}(\Phi) < ARL_0$

Table 5.2 gives the percentages of in-control ARLs in $\bar{d}$ chart compared to Hotelling's T^2 chart which fall into equal, better and worst performances categories, when $m = 20, 50, 70$. In general, $\bar{d}$ chart showed approximately 40% equivalent performance to T^2 chart when the number of subgroups $m = 50$ and above. From Table 5.3, the S_d chart showed equal proportion (40%) of better and worse performances when compared to $|S|$ chart.

The performance of the difference charts is further investigated using several out-of-control scenarios when the parameters are estimated from Phase I data. The case

Table 5.2: Performances of $\bar{d}$ chart based on the percentages in-control ARLs compared to Hotelling's T^2 chart. The control limits were obtained from estimated parameters from limited Phase I data of subgroups m of size $n = 10$

Performance of $\bar{d}$ chart	Compared to T^2 chart $m = 20$	$m = 50$	$m = 70$
Better	36.3%	14.5%	17.0%
Equal	7.8%	38.8%	36.9%
Worse	55.9%	46.7%	46.1%

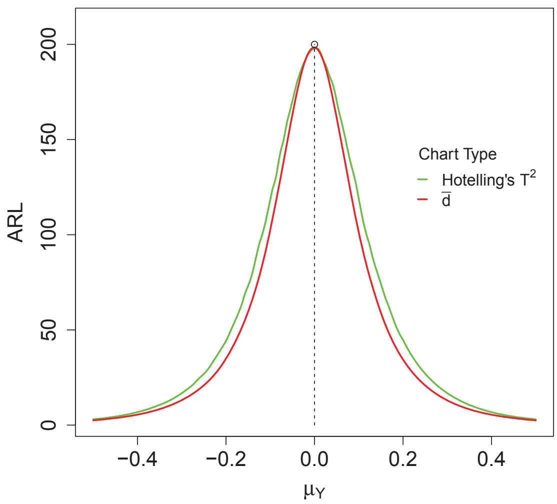
Table 5.3: Performances of S_d chart based on the percentages in-control ARLs compared to $|S|$ chart. The control limits were obtained from estimated parameters from limited Phase I data of subgroups m of size $n = 10$

Performance of S_d chart	Compared to $\|S\|$ chart $m = 20$	$m = 50$	$m = 70$
Better	41.20%	33.65%	34.00%
Equal	19.05%	21.15%	24.2%
Worse	39.75%	45.2%	41.80%

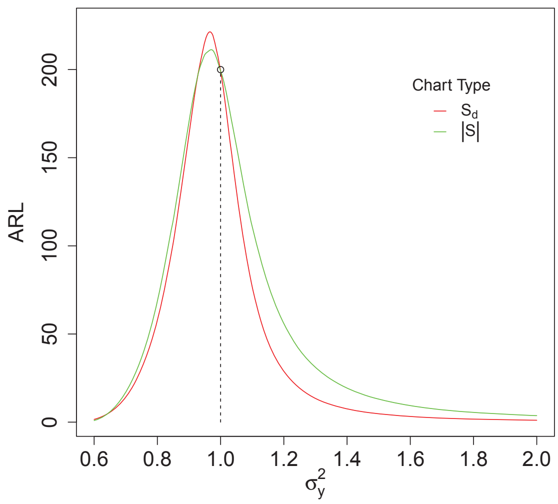
where subgroup size $m = 50$, $n = 10$ and $\alpha = 0.005$ are selected. When comparing the performances of the charts, all should exhibit the same in-control ARLs. This has been achieved by adjusting the control limits to get a common in-control ARL of 200. For Hotelling's T^2 chart, the α is adjusted and a simulation approach was followed for other charts. From the out-of-control cases given in subsection 5.3.1, three cases where **Case I: Shifts in one mean**, **Case 4: Shifts in individual variance** and **Case 7: Shift in correlation coefficient** are chosen to discuss the performances of the difference charts for limited Phase I data. Figures 5.8a, 5.8b and 5.9 show that the $\bar{d}$ chart performs well compared to the Hotelling's T^2 chart: the S_d chart performs better than the $|S|$ chart for fixed shifts in σ_Y^2 and ρ.

The difference charts and other charts that have used for the comparison show relatively large variability in the in-control run length distribution when $m = 20$. At least

Figure 5.8: ARLs for (a) shifts in μ_Y (b) shifts in σ_Y^2 when the parameters are estimated from Phase I data

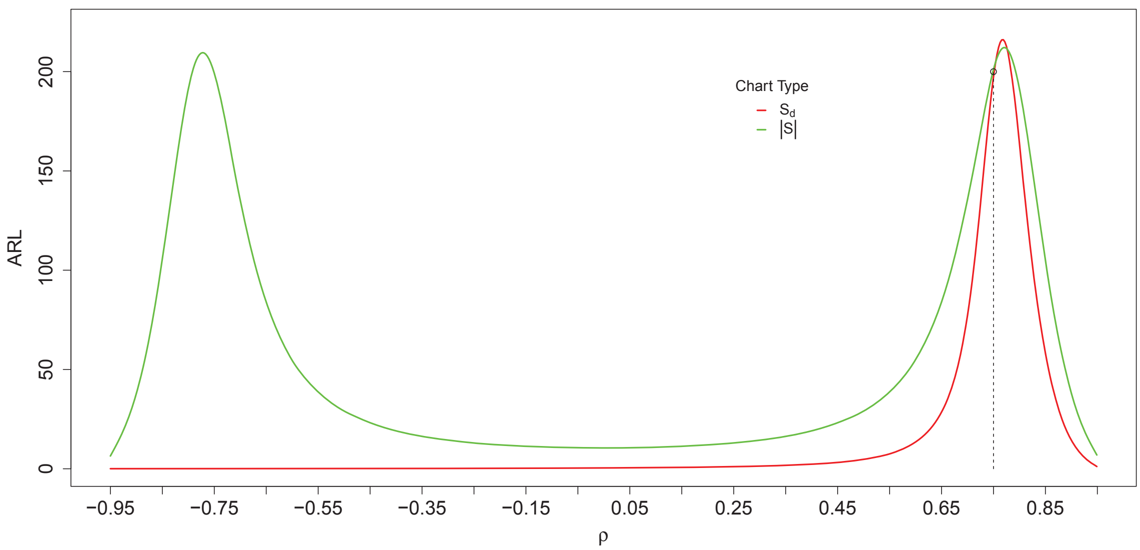


(a)



(b)

Figure 5.9: ARLs for shifts in ρ when the parameters are estimated from Phase I data



$m = 50$ subgroups to be used for Phase I is suggested. At this point, the difference charts are found to be superior to their common counterparts for detecting shifts in the process parameters.

5.3.3 Discussion of RLs properties using inverse-Wishart prior

From the classical point of view, the parameters of the common cause distribution are fixed but unknown. Deming's view that "no process is steady" is a contradiction to the fixed parameters assumption, see Woodall (2000). From a Bayesian point of view, "no process is steady" is not a contradiction because the parameters are treated as random variables. The classical approach commonly models the production process as a fixed population under common causes. On the other hand, the Bayesian approach treats the underlying process as a superpopulation, and the process only becomes a fixed population over the short horizon. The assumed prior distribution allows for process uncertainty in a long horizon. In a broader risk perspective, Bayesian methods allow for epistemic uncertainties (such as difficulty in taking random samples etc.). The aleatory approach of assuming that the in-control process is always steady can underestimate the overall risk which may have a significant impact on some applications, such as stock market data. As a consequence, the RL performance of a control chart can differ depending on whether the shifts in the parameters are fixed or not. In subsections 5.3.1 and 5.3.2, known fixed shifts are considered in the process parameters. By using an inverse-Wishart (IW) prior distribution, the case of unknown shifts occurring in a random manner was covered. The IW distribution is used to account for out-of-control parameter uncertainty in $\hat{\Sigma}$ when assessing the performance of the $\bar{d}$ and S_d charts.

Duncan (1971) employed the Bayesian methodology for the design of the $\bar{X}$ chart. The IW distribution is commonly used as the conjugate prior for the covariance matrix, for example Triantafyllopoulos (2011) and Tan and Shi (2012) have used this approach in the context of multivariate control charts. The IW distribution for the covariance matrix Σ is given as $\Sigma \sim IW(\nu, \Lambda)$ where Λ is a positive d dimensional matrix and ν represents the degrees of freedom; then $\mathrm{E}(\Sigma) = \Sigma_0 = \frac{\Lambda}{\nu - d - 1}$.

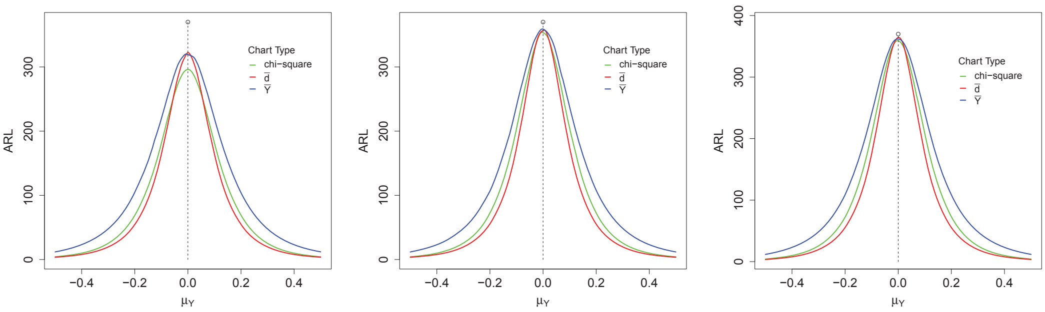
The IW prior distribution approach allows for uncertainty to be examined through parameter ν (also known as the precision parameter). The classical case can be obtained when ν becomes large. In contrast, the sampling variability is large when ν is small. Altering ν means we do not have direct control over the actual rate of occurrence being imposed.

The ARL performance of a control chart can differ depending on whether the parameters are known or unknown and fixed or not. By using the inverse Wishart prior distribution, we cover the cases of unknown and unfixed parameters. The actual design of the difference charts is left for future work. This approach is implemented as follows: a realization of the covariance matrix is obtained under the IW prior for the assumed ν and given $\boldsymbol{\Sigma}$. The `riwish` function in R: MCMCPack (Martin et al., 2011) is used to obtain an unscaled realization of $\boldsymbol{\Sigma_W}$ and then scaled up as $\boldsymbol{\nu}\ \boldsymbol{\Sigma_W}$. The realised covariance matrix and the mean vector were used to generate a single bivariate normal sample in *Step* 2 of Algorithm 5.1. In this situation, the covariance matrix for generating Phase II data is not explicitly given and differ among each iteration when generating data for subgroups. For this analysis, an assumption made that the Phase I parameters are given.

The out-of-control ARLs performances of $\bar{d}$ and S_d charts are investigated under inverse-Wishart prior for all the out-of-control situations which have been considered in Section 5.3.1. Results are presented only for the **Case 1** and **Case 4** but the similar results were found for other situations.

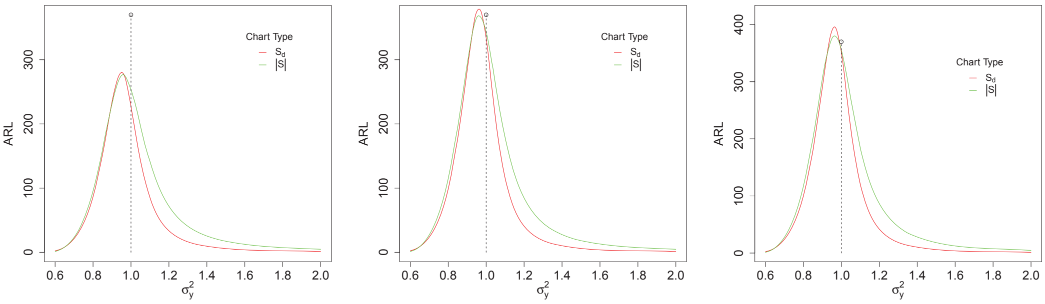
Figures 5.10a, 5.10b and 5.10c show **Case 1: shift in μ_Y**, under IW prior for $\nu = 100, 500$ and $\nu = 1000$ respectively. The uncertainty in the shift level is controlled by ν which is inversely related to the degree of uncertainty. Figure 5.10c is based on a higher precision

Figure 5.10: (a) ARLs for shifts in μ_Y under a Wishart prior for $\nu = 100$ and (b) ARLs for shifts in μ_Y under a Wishart prior for $\nu = 500$ (c) ARLs for shifts in μ_Y under a Wishart prior for $\nu = 1000$. ($\mu_X = 0$, $\sigma_X^2 = 1, \sigma_Y^2 = 1$ and $\rho = 0.75$)



(a) **(b)** **(c)**

Figure 5.11: (a) ARLs for shifts in σ_Y^2 under a Wishart prior for $\nu = 100$ and (b) ARLs for shifts in σ_Y^2 under a Wishart prior for $\nu = 500$ (c) ARLs for shifts in σ_Y^2 under a Wishart prior for $\nu = 1000$. ($\mu_X = 0, \mu_Y = 0$ $\sigma_X^2 = 1$ and $\rho = 0.75$)



(a) **(b)** **(c)**

level of $\nu = 1000$ and hence the ARLs are only slightly smaller when compared to the case of having no prior in Figure 5.1. Similar results were found for the S_d and $|S|$ charts (see Figures 5.11a,5.11b and 5.11c for $\nu = 100, 500, 1000$, respectively). The main objective of the IW approach is to validate the performances of the new charting procedures under stringent but more realistic conditions.

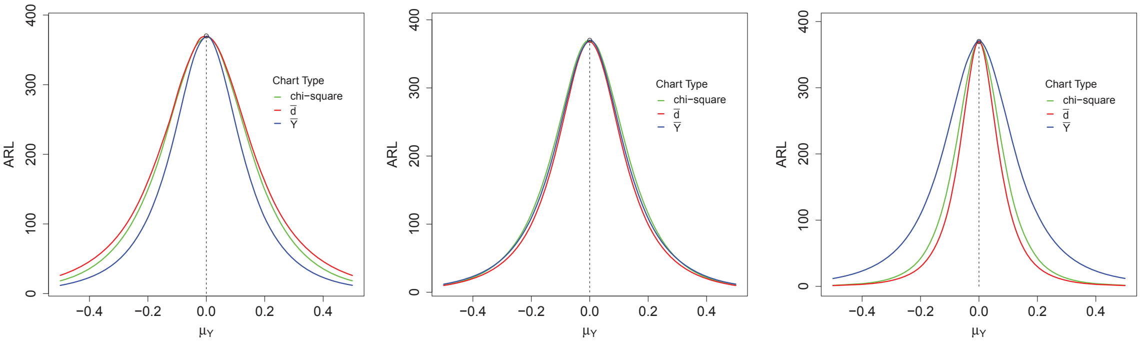
In this subsection, chart performances are evaluated for the situation where actual shifts are unknown. The difference charts were found performed well in comparison with the chi-square and $|S|$ charts.

5.3.4 Other remarks

Interpretation of out-of-control signals in multivariate control charts have been studied by many researchers. Grabov and Ingman (1996) outlined that the main disadvantage of charts such as Hotelling's T^2 chart is their inability to identify the individual variable causing the out-of-control signal. For multivariate control chart procedures, comparisons of joint versus simultaneous monitoring strategies are made in Khoo (2004) and McCracken and Chakraborti (2013). In the case of simultaneous monitoring, the covariance structure is not used for the determination of control limits. In the case of joint monitoring, identification of the variable causing the signal is harder. The difference chart procedure is a compromise between the joint and simultaneous monitoring strategies. The S_d chart responds to the shifts in the co-variance matrix but the identification of cause variable is not possible; the user may need to use individual S charts in conjunction with the S_d chart.

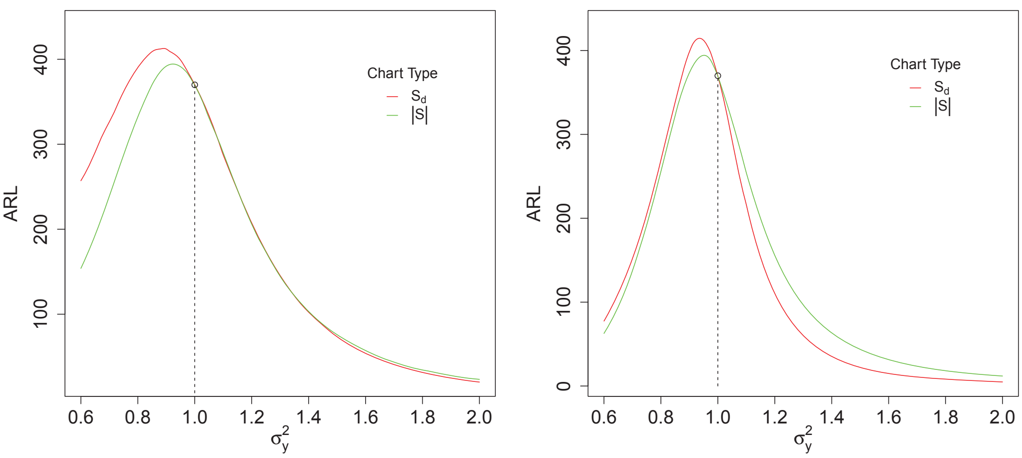
The effect of correlation on the performance of the difference charts has been investigated. The difference charts perform equivalently or better than the other bivariate charts we investigated; the dominance of the difference charts over the other bivariate charts is more pronounced when correlation is strong (see Figures 5.12 and 5.13). If the correlation is not significant enough, not only the difference charts but also other bivariate charts are not required; for those situations, the charts for individual variables should be preferred.

Figure 5.12: ARLs for shifts in μ_Y for $\rho = 0.15, \rho = 0.55$ and $\rho = 0.85$



(a) **(b)** **(c)**

Figure 5.13: ARLs for fixed shifts in σ_Y^2 for $\rho = 0.15$ and $\rho = 0.55$



(a) **(b)**

5.4 A finance application

The previously-reported applications of the $\bar{d}$ chart are limited to manufacturing processes; see Grubbs (1946); Ott (1947) and Tracy et al. (1995). Time series of paired variables are commonly studied in the finance literature. Investors are concerned with the changes in the returns of a stock relative to the overall market performance. Stock returns are usually correlated with market returns and hence investors are interested in monitoring how well a particular stock is performing relative to the market or its peers in the same sector.

The *beta* measure of a stock reveals the degree of association between its price changes and the market index; (see Graham and Smart, 2011, pp.211-218). Monitoring the shifts in mean and variance (volatility) of the differences of the returns of an asset and the market is easier to understand for practitioners than interpreting the shifts in the risk measures such as correlation or a stock's *beta*. We cannot review the cause of shifts from the changes in correlation coefficient or a stock's *beta*.

Beta is the popular measure of an asset's risk with a portfolio. In the CAPM, *beta* of a stock measures the risk of that particular asset relative to the market and controlling *beta* is influential on the rational investors' expected returns (see Campbell and Vuolteenaho, 2004). The well-known market indices such as the S&P 500 or the Nasdaq index can be used as a benchmark for the market. The slope of the linear regression line between returns for a given asset and returns for a market index, is used as an estimate for *beta*. Correlation coefficient and *beta* are both time-variant (see Gilbert et al., 2014; Longin and Solnik, 1995; Krishnan et al., 2009; Ghysels, 1998; Harvey, 1991).

The current literature has focused on the detection of changes and monitoring (mean vector or covariance) relative to a chosen model for a financial time series, see for example Garthoff et al. (2014a), Garthoff et al. (2014b). Bodnar and Schmid (2009) explained the limitations of applying those sequential procedures to financial data. The main problem is the uncertainty in selecting a suitable model due to special cause contamination. Aue et al. (2012), Chochola et al. (2013) and Chochola et al. (2014) applied a change detection procedure initially proposed by Chu et al. (1996) for the portfolio *Beta*. Wied and Galeano (2013) used the same approach for the correlation coefficient. In change point detection procedures, the particular statistic is measured using historical data to detect the change point as new data becomes available. This procedure does not consider separation of special and common cause variations.

Golosnoy (2016) has proposed a sequential monitoring procedure using the Shewhart control chart for a *Beta* of a single portfolio and the Hotelling T^2 chart for multiple *Beta* values of a set of portfolios. This reference shows the growing need for monitoring procedures for *Beta* considering both Phase I and Phase II analysis. A control chart procedure for *Beta* or correlation coefficient monitoring will require further investigation due to a possible lack of stability over short series of data.

The mean and variance of the difference series of stock and market returns incorporate individual means, variances and covariance between the two series. This Chapter showed that difference charts are more effective than the Hotelling T^2 chart and individual Shewhart charts for bivariate data. Monitoring of mean and standard deviation shifts in the difference series would be more efficient than monitoring of existing risk measures such as the correlation coefficient and the stock's *Beta*. The difference charts provide more information because the $\bar{d}$ chart responds to the mean shifts and the S_d chart reacts for shifts in individual volatilities or the correlation coefficient.

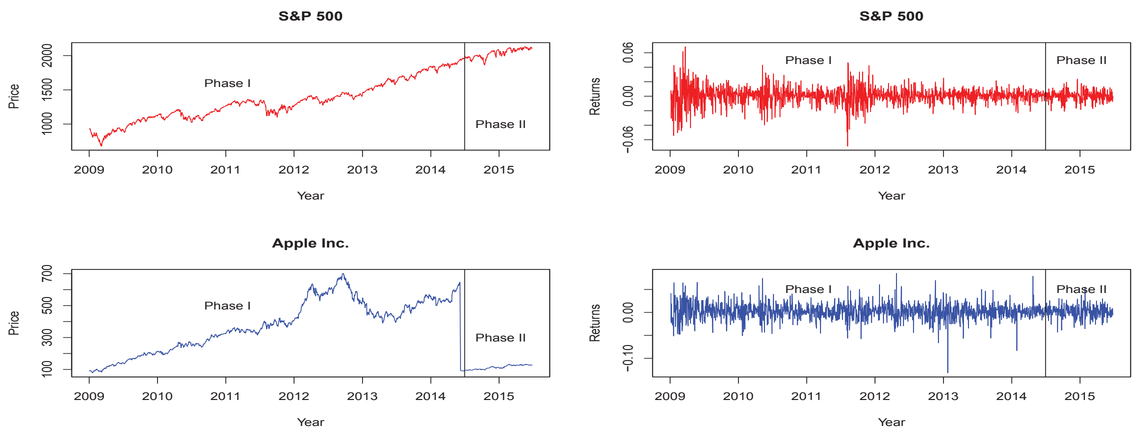
Daily stock returns were considered of Apple Inc. (Y) and the S&P 500 index (X) as paired variables in order to demonstrate the application of the $\bar{d}$ and S_d charts and the results are provided below. The main aim is to analyse the movements of a particular stock relative to market movements. The S&P 500 index was chosen as a proxy for the stock market. The daily log returns from June 2009 to June 2014 for the Phase I analysis and from

June 2014 to June 2015 for the Phase II analysis were used. Figures 5.14a and 5.14b display the price and return series data considered in this analysis without the data pertaining to a known special cause event (a seven-for-one stock split in June 2014, which resulted in a sudden drop in the prices, see Figure 5.14a). Fortnightly subgroups were used (10 trading days approximately) but users can select the desired and appropriate subgroup size. The modified stepwise procedure of robust estimation of the mean and standard deviation was followed from Section 3.3.

Figures 5.15a and 5.16a give the $\bar{d}$ and Hotelling's T^2 charts respectively for monitoring the mean of the differences in Phase II returns between Apple Inc. and the S&P 500. Both charts do not issue any signals for the Phase II period but this is not the case with the S_d chart shown in Figure 5.15b. The S_d chart monitors Apple Inc.'s measured volatility in relation to the market over a two-week period. Figure 5.16b shows the $|S|$ chart for Phase II data but yielded an excessive number of signals which are difficult to interpret in practice. The S_d chart yielded only two signals and these are easy to interpret. The S_d chart clearly identified the increased volatility in prices which is of interest to short-term investors and traders.

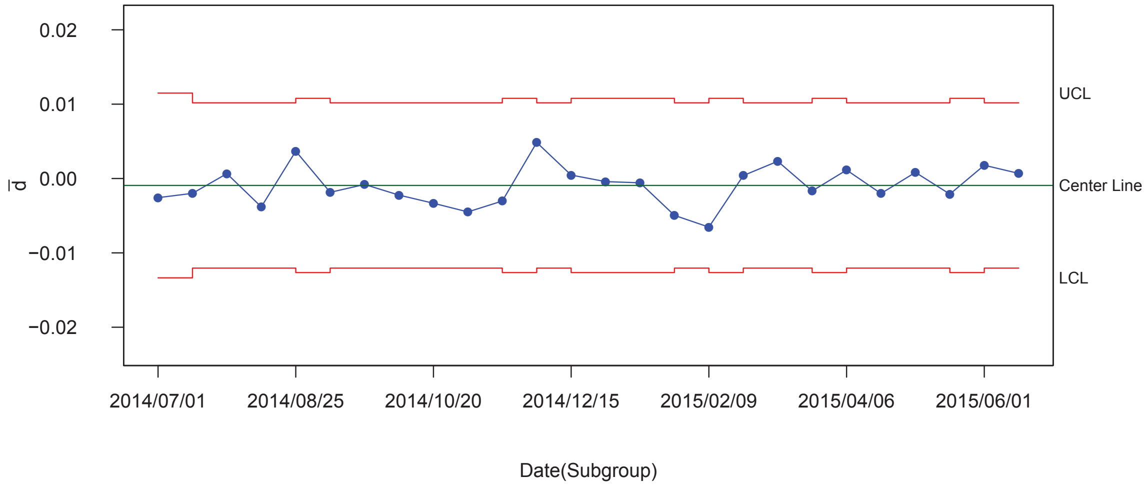
This example demonstrated that the difference charts performed better than the other charts for monitoring mean vector and covariance matrix. Trading frequency affects market strategies and the associated risk. Most fund managers use sophisticated tools to assist their investment decisions. Difference charts can be incorporated with those tools. The use of tools developed incorporating quality management principles in financial analysis have the ability to improve those investment decisions.

Figure 5.14: (a) daily price series and (b) log returns

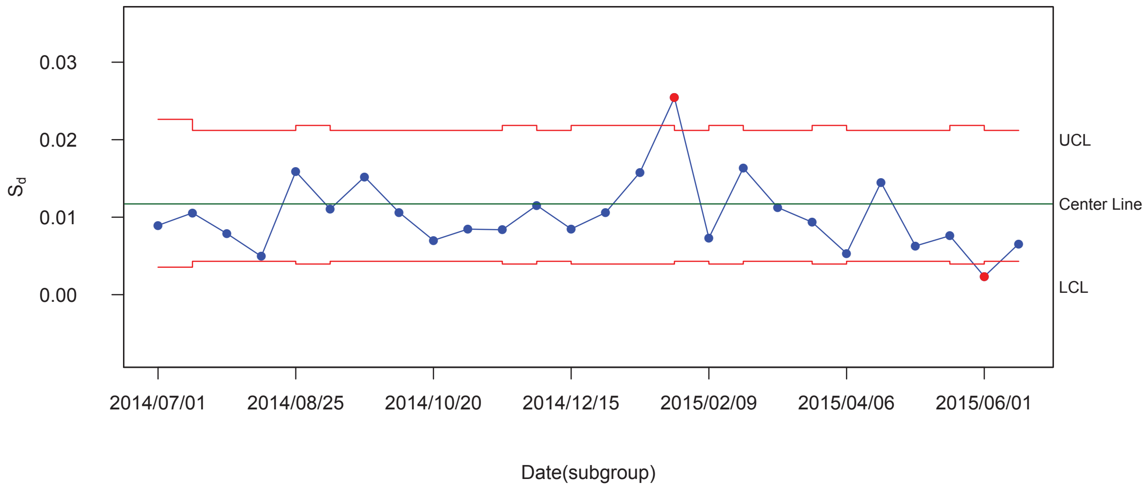


(a) **(b)**

Figure 5.15: Phase II (a) $\bar{d}$ chart and (b) S_d chart

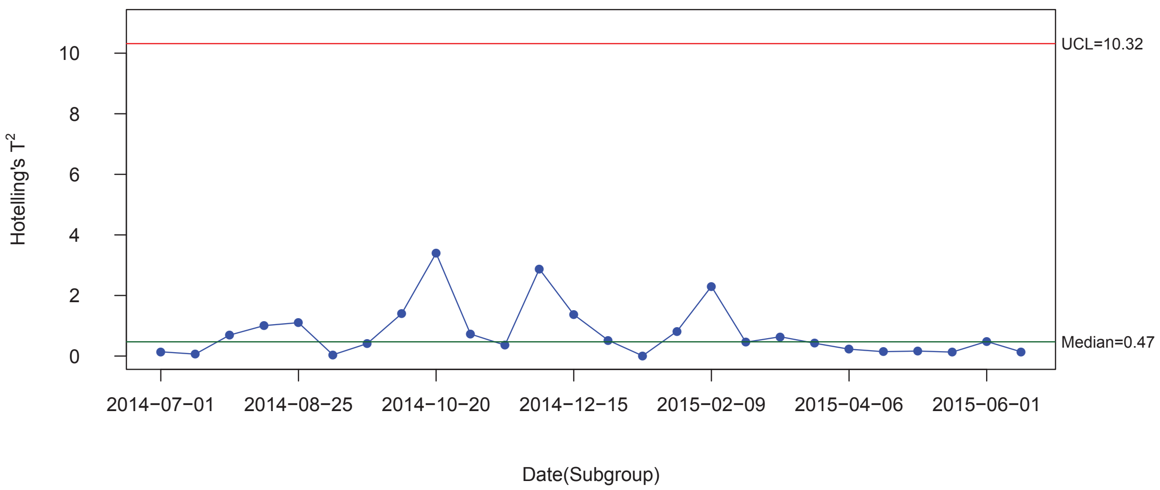


(a)

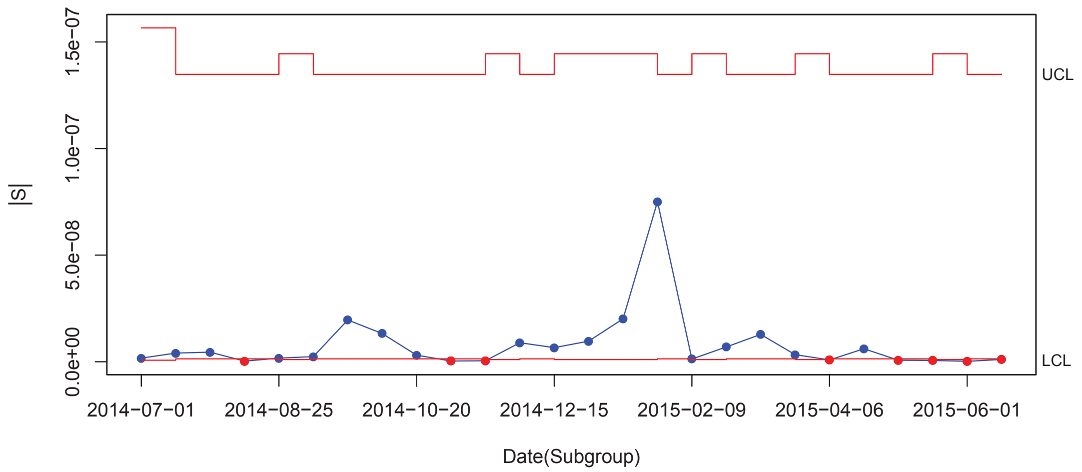


(b)

Figure 5.16: (a) Hotelling's T^2 chart and (b) $|S|$ chart



(a)



(b)

5.5 Summary

Difference control charts are useful for monitoring a correlated pair of variables. Findings from this chapter show that the $\bar{d}$ chart is powerful for monitoring the changes in the means and the S_d chart is suitable for monitoring changes in the covariance structure.

Based on an extensive simulation study, it is found that the $\bar{d}$ and S_d charts perform better than existing bivariate control charts for detecting shifts in mean and variance/covariance respectively when standards are known. The difference charts also performed well compared to common alternatives when the standards arising from a limited amount of Phase I data are unknown.

The S_d chart introduced in this chapter is useful for detecting changes in the correlation as well as in the variance structure. It is therefore particularly useful for monitoring the volatility in stock returns.

The difference charts will be used to identify common and special causes periods in Chapter 6, because the distribution of difference series preserves the joint relationship between asset and market and then the bivariate normality is discussed.

Chapter 6

Bivariate co-moments in stock returns

This chapter investigates the behaviour of co-moments namely co-variance, co-skewness and co-kurtosis in total and common cause periods to assess *Proposition* 2. This study shows that the symmetry between the co-moments is theoretically valid for a standard bivariate normal distribution and hence this approach has the potential for assessing the co-moments in real data.

The difference chart procedure from Chapter 5 is used to partition the difference series of returns of a selected stock and the S&P 500 index. The S&P 500 index is used as a proxy for the market returns.

Findings in this chapter are presented in such a way that market participants can use co-moments in the extended forms of capital asset pricing models by addressing the *Issue* 2. In addition, this chapter proposes new alternative definitions for co-moments based on the differences in all non-matching pairs of observations in the two series. Using Monte Carlo simulation, the power of the newly defined co-moments is examined and their advantages are explained later in this chapter.

6.1 Introduction

Co-moments are largely used in financial applications when compared to other applications. Various interpretations of co-moments are made in the finance literature. For example, "co-skewness measures the symmetry of an asset returns' distribution in relation to the market returns' distribution" (Kostakis, 2009, p.464); "the co-skewness provides information on the symmetry of the distribution of X" and "the co-kurtosis provides information on the thickness of the tails of the distribution of X", where X is a N-dimensional random variable (Meucci, 2009, p.59); "co-kurtosis measures the likelihood that extreme returns jointly occur in a given asset and in the market" (Ranaldo and Favre, 2005b, p.2).

The mathematical way of dealing with skewness and kurtosis for asset allocation is to consider the first four moment expansion of the Taylor series as an approximation of the expected utility function, (see Jondeau and Rockinger, 2006, pp. 1-2). The earliest mathematical definitions for systematic co-skewness and co-kurtosis can be found in Kraus and Litzenberger (1976, p.1088) and Fang and Lai (1997, p.298) respectively. A list of various definitions for co-skewness is given in Chen (2016, pp.208-209). On the other hand, by first principles, there are two non-trivial measures for co-skewness: $CS(X,Y^2), CS(X^2,Y)$ and three measures for co-kurtosis : $CK(X,Y^3), CK(X^2,Y^2), CK(X^3,Y)$, where X and Y are bivariate random variables.

Chen (2016) has reported the issue of choosing one measure from the multiple measures for co-moments in the extended CAPM; however in this study, the symmetric behaviour among the multiple measures is employed for co-skewness and co-kurtosis. In Section 6.2, symmetric behaviour of co-skewness and co-kurtosis is proved mathematically when the two series follow a standard bivariate normal distribution. This property is then used to benchmark the observed pattern in the real data.

Chapters 1, 2 and 4 focussed on the use of Shewhart principles to understand process variations as appropriate for financial series. According to Shewhart's postulates, future variability is unquantifiable and fitted models cannot be adjusted for such unpredictable special cause events. In Section 6.3, the difference series of stock and market returns (by following Chapter 5) is partitioned into common and special causes. The co-moments are then examined in the total and common cause periods. The objective is to demonstrate the effect of unforeseen market events on the co-moments in the partitioned data. This would help market researchers to employ alternative approaches or to adjust conventional models to accommodate the first four moments and co-moments related variations seen in the data.

In this chapter, correlation coefficient between an individual asset and the market is also investigated in the partitioned data. Markowitz (1952) has described the effect of the co-movement between individual assets to the risk of a portfolio using the correlation coefficient. However, Damghani et al. (2012) and the references therein addressed the issues of measuring correlation relating to non-linearity and properties such as skewness and kurtosis in return series. There is a need for clarifying the validity of correlation coefficient in this context. When the correlation between asset returns (it can returns of a market index or the entire portfolio) is large the diversification benefits for the investors in their portfolios is reduced implying that correlation can be used as a diversification risk measure (Krishnan et al., 2009). Moreover many authors including Driessen et al. (2009); Krishnan et al. (2009) have reported the time-varying nature of correlations and unexpected increase in stock returns during periods of market crisis.

Co-moments are not uniquely defined and no method for testing the significance of excess co-moments has been reported in the literature. Richardson and Smith (1993) tested excess co-skewness and co-kurtosis but the sample measures for these co-moments depend on the unknown sample correlation coefficient. Section 6.2 will show that when the two series follow the standard bivariate normal distribution, co-skewness is equal to zero. As a consequence, the null hypothesis of zero co-skewness can be examined (see

Harvey and Siddique, 2000, p.1278) but the co-kurtosis depends on the true correlation coefficient.

When the true correlation coefficient is unknown, the testing of the significance of excess co-kurtosis using available sample estimates is not straightforward. Since the sample correlation coefficient does not capture the non-linearity in the data, an explicit method is therefore needed to test the significance of the excess co-kurtosis. Testing the significance of excess co-moments is not of primary importance of this study but it is believed that this analysis will contribute to future research on this topic.

By definition, co-variance measures how two variables move together or the association between them. The sample covariance is calculated by the average of the product of the mean deviations of paired variables. Without referring to the means, the products of all possible bivariate differences can be used to estimate the co-moments. For example Zhang et al. (2012) proposed a new estimate based on all the differences in 'matching pairs' of observations for the co-variance.

New sample measures are therefore defined in this study for co-variance, co-skewness and co-kurtosis which are based on the differences of all non-matching pairs of observations for the two series: $(X,Y) = (x_i, y_i), i = 1,...,N$, i.e. the new estimates are based on the $(x_i - y_j)$, where $i \neq j$ and $x_i \in X, y_j \in Y$. The behaviour of these new estimates is opposite to that of existing co-moments based on matching pairs. The statistical power and false alarm rate between these new measures and existing co-moments estimates are compared using a Monte Carlo experiment. A detailed discussion of this approach is given in Section 6.4. This study is limited to the numerical analysis of proposed measures. Section 6.5 concludes the chapter.

6.2 Theory of co-skewness and co-kurtosis

In this section, theoretical relationships are derived for co-skewness and co-kurtosis when the (X, Y) follows a bivariate normal distribution. The definitions are given in equation 6.1.

$$\begin{aligned}
\text{co-skewness:}\quad & CS(X,Y^2) = \mathrm{E}\left[(X-\mathrm{E}[X])\,(Y-\mathrm{E}[Y])^2\right]\\
& \text{or}\\
& CS(X^2,Y) = \mathrm{E}\left[(X-\mathrm{E}[X])^2\,(Y-\mathrm{E}[Y])\right]\\
\text{co-kurtosis:}\quad & CK(X,Y^3) = \mathrm{E}\left[(X-\mathrm{E}[X])\,(Y-\mathrm{E}[Y])^3\right]\\
& \text{or}\\
& CK(X^2,Y^2) = \mathrm{E}\left[(X-\mathrm{E}[X])^2\,(Y-\mathrm{E}[Y])^2\right]\\
& \text{or}\\
& CK(X^3,Y) = \mathrm{E}\left[(X-\mathrm{E}[X])^3\,(Y-\mathrm{E}[Y])\right]
\end{aligned} \tag{6.1}$$

Consider the bivariate normal density,

$$f(x,y) = \frac{1}{2\phi\sigma_X\sigma_Y\sqrt{1-\rho^2}}e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_X}{\sigma_X}\right)^2-\rho\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right)+\left(\frac{y-\mu_Y}{\sigma_Y}\right)^2\right]}$$

and its moment generation function (mgf) to derive the equation for co-moments when $\mu_X = \mu_Y = 0$. This approach was initially proposed in Richardson and Smith (1993).

$$M_{X,Y}(t_1,t_2) = exp\left(\mu_x t_1 + \mu_y t_2 + \frac{\sigma_X^2 t_1^2 + 2\rho\sigma_X\sigma_Y t_1 t_2 + \sigma_Y^2 t_2^2}{2}\right) \tag{6.2}$$

Then,

$$\mathrm{E}\left[X^m Y^n\right] = \frac{\partial^{m+n}\left[M_{X,Y}(0,0)\right]}{\partial t_1^m \partial t_2^n} \tag{6.3}$$

For the case where, $\mu_X = \mu_Y = 0$, from the equation 6.1, co-skewness: $CS(X,Y^2) = \mathrm{E}\left[XY^2\right]$ and $CS(X^2,Y) = \mathrm{E}\left[X^2Y\right]$. So,

$$\mathrm{E}\left[XY^2\right] = \frac{\partial^3\left[M_{X,Y}(0,0)\right]}{\partial t_1 \partial t_2^2} \text{ and } \mathrm{E}\left[X^2Y\right] = \frac{\partial^3\left[M_{X,Y}(0,0)\right]}{\partial t_1^2 \partial t_2} \tag{6.4}$$

$$\begin{aligned}
\frac{\partial^3\left[M_{X,Y}(0,0)\right]}{\partial t_1 \partial t_2^2} &= \mu_X\mu_Y^2 + 2\rho\mu_Y\sigma_X\sigma_Y + \mu_X\sigma_Y^2 \\
\frac{\partial^3\left[M_{X,Y}(0,0)\right]}{\partial t_1^2 \partial t_2} &= \mu_X^2\mu_Y + 2\rho\mu_X\sigma_X\sigma_Y + \mu_Y\sigma_X^2
\end{aligned}$$

Hence, $CS(X,Y^2) = CS(X^2,Y) = 0$ for $\mu_X = \mu_Y = 0$ and $\sigma_X^2 = \sigma_Y^2 = 1$. Similarly, $CK(X,Y^3) = \frac{\mathrm{E}[XY^3]}{\sigma_X\sigma_Y^3}$, $CK(X^2,Y^2) = \frac{\mathrm{E}[X^2Y^2]}{\sigma_X\sigma_Y^3}$ and $CK(X^3,Y) = \frac{\mathrm{E}[X^3Y]}{\sigma_X\sigma_Y^3}$, when $\mu_X = \mu_Y = 0$. Considering the partial derivatives of mgf we can show that,

$$\begin{aligned}
\mathrm{E}\left[XY^3\right] &= \mu_X\mu_Y^3 + 3\mu_X\mu_Y\sigma_Y^2 + 3\rho\mu_Y^2\sigma_X\sigma_Y + 3\rho\sigma_X\sigma_Y^3 \\
\mathrm{E}\left[X^2Y^2\right] &= \mu_X^2\mu_Y^2 + \mu_X^2\sigma_Y^2 + 4\rho\mu_X\mu_Y\sigma_X\sigma_Y + \mu_y^2\sigma_X^2 + 2\rho^2\sigma_X^2\sigma_Y^2 + \sigma_X^2\sigma_Y^2 \\
\mathrm{E}\left[X^3Y\right] &= \mu_X^3\mu_Y + 3\mu_X\mu_Y\sigma_X^2 + 3\rho\mu_X^2\sigma_X\sigma_Y + 3\rho\sigma_X^3\sigma_Y
\end{aligned} \tag{6.5}$$

From equation 6.5, when $\mu_X = \mu_Y = 0$ and $\sigma_X^2 = \sigma_Y^2 = 1$:

$$\begin{aligned}
CK(X,Y^3) = CK(X^3,Y) &= 3\rho \\
CK(X^2,Y^2) &= 2\rho^2 + 1
\end{aligned} \tag{6.6}$$

The results show that the co-kurtosis depends only on the ρ for $\mu_X = \mu_Y = 0$ and $\sigma_X^2 = \sigma_Y^2 = 1$. The alternative measures for co-skewness $\left(CS(X, Y^2), CS(X^2, Y)\right)$ and co-kurtosis $\left(CK(X, Y^3), CK(X^3, Y)\right)$ are symmetrical for the standard bivariate normal distribution.

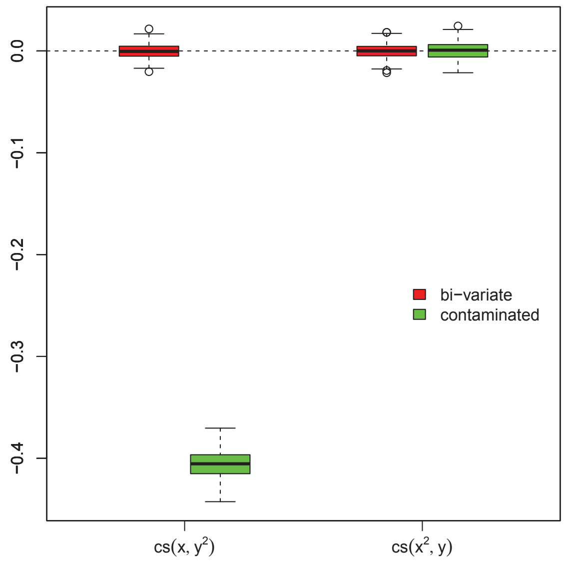
6.2.1 Sample co-moments

In this section, the sample co-moments are investigated using random bivariate data. The aim is to find how co-moments affect by the outliers. Let us consider the sample co-moments for all the possible measures of co-skewness and co-kurtosis,

$$\begin{aligned}
cs(x, y^2) &= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})\left(y_i - \bar{y}\right)^2 \\
cs(x^2, y) &= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\left(y_i - \bar{y}\right) \\
ck(x, y^3) &= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})\left(y_i - \bar{y}\right)^3 \\
ck(x^2, y^2) &= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\left(y_i - \bar{y}\right)^2 \\
ck(x^3, y) &= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3\left(y_i - \bar{y}\right)
\end{aligned} \tag{6.7}$$

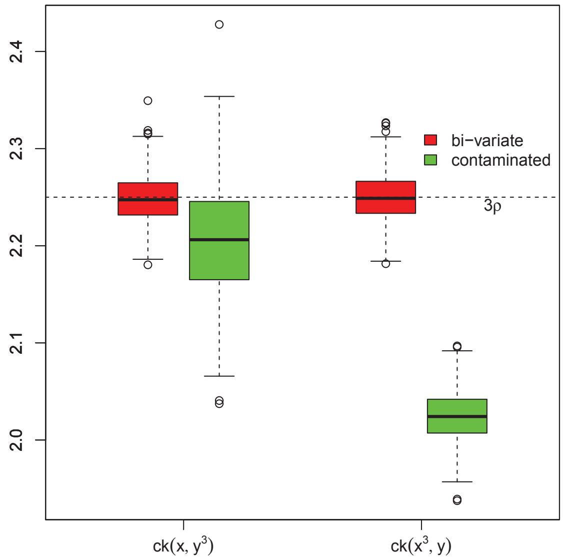
Bi-variate normal random data was generated from the MASS: Venables and Ripley (2002) R-package for $\mu_X = \mu_Y = 0$ and $\sigma_X^2 = \sigma_Y^2 = 1$. Built-in R functions for sample co-moments were taken from the “PerformanceAnalytics" (Carl and Peterson, 2014) R-package. Figures 6.1 and 6.2 show the symmetric behaviour in two co-skewness measures; $cs(x, y^2)$, $cs(x^2, y)$ and in two co-kurtosis measures; $ck(x, y^3)$, $ck(x^3, y)$ when data follows a standard bivariate normal distribution. The Y series is then contaminated by replacing 10% of data in the original series with randomly generated unusual values. This was done creating a random index of size is 10% of the length of Y series, then the corresponding values in the Y series is replaced by the randomly generated unusual values (between 0

Figure 6.1: $cs(x, y^2)$ vs $cs(x^2, y)$ for standard bivariate normal data when $\mu_X = 0$, $\mu_Y = 0$, $\sigma_X^2 = 1$, $\sigma_Y^2 = 1$, $\rho = 0.75$ and Y series with 10% contamination



and 10). For contaminated bivariate data these figures also show a large inconsistency between the pair of co-skewness and co-kurtosis measures. This property is used to identify the effect of contamination or special cause periods in real data in Section 6.3.

Figure 6.2: $ck(x, y^3)$ vs $ck(x^3, y)$ for standard bivariate normal data when $\mu_X = 0$, $\mu_Y = 0$, $\sigma_X^2 = 1$, $\sigma_Y^2 = 1$, $\rho = 0.75$ and Y series with 10% contamination



6.3 Empirical analysis

The co-moments were investigated between the returns of S&P 500 stocks and the S&P 500 index itself. Additional details for the empirical data were given in Section 1.4. In practice, investors usually monitor the price fluctuations in a particular asset or a portfolio relative to the well known market index. The partition procedure for common and special cause periods which is based on an individual series can be criticised. Subsequently, the partition of special and common causes is based on the difference series: $d = X - Y$, where X represents the returns of S&P 500 index and Y denotes the returns of a particular asset.

Chapter 5 has shown that difference charts (bivariate charts for paired differences) possess certain advantages for financial applications over the other bivariate charts. In this section, the control limits from the S_d chart are used as a decision criteria for the separation of special and common causes. The control limits for the S_d charts are the same as the limits of traditional (Shewhart) S chart: for the S_d chart, the probability limits can be fixed as $\frac{\bar{S}_d}{c_4(n)}\sqrt{\frac{\chi^2_{0.99865}}{n-1}}$ and $\frac{\bar{S}_d}{c_4(n)}\sqrt{\frac{\chi^2_{0.00135}}{n-1}}$, where $\bar{S}_d$ is the average subgroup standard deviation of the differences (more details were given in Chapter 5). The stepwise robust chart procedure given in Section 3.3 is used for establishing the control limits for the S_d chart. Using this procedure, contaminated time periods are identified in the difference series. Original returns are then extracted from the X and Y series which match the time index of common and special cause periods in the d series.

The table of sample moments (mean, standard deviation, skewness and kurtosis) for the returns of S&P 500 index and selected stocks is reproduced again (see Table 6.1) so that the reader can compare the behaviours of moments and co-moments. Table 6.2 gives the sample co-variance $(s(x, y))$, co-skewness $\left(cs(x, y^2), cs(x^2, y)\right)$ and co-kurtosis $\left(ck(x, y^3), ck(x^3, y)\right)$ values for selected stocks. When calculating the co-moments, the standardised values of the X and Y series are used.

For standard bivariate normal data or under common cause variations, zero co-skewness and equality of two co-kurtosis measures $\left(ck(x, y^3), ck(x^3, y)\right)$ hold. If not a departure from bivariate normality is indicated. Table 6.2 shows non-zero co-skewness and unequal co-kurtosis values. From both Tables 6.1 and 6.2, the stocks which show 'excess' moments also show the 'mismatching' co-moments. Table 6.3 shows the approximately equal $ck(x, y^3)$ and $ck(x^3, y)$ values in common cause periods but non-zero co-skewness.

Figure 6.3 shows the sample correlation coefficient for total and common cause periods. The deviation from the expected $X = Y$ pattern is strong and the stocks which are in the top-left corner of the figure showed a large excess skewness and kurtosis in the total series, for example, Torchmark Corp. (Ticker symbol: TMK) has skewness: –30.08 and kurtosis: 937.47 in the total series. Clearly these return series were contaminated

Table 6.1: The first four sample moments in the total and common cause data of the return series for S&P 500 index and selected nine stocks: January, 2013 to December, 2015

		Total data				Common cause data			
Ticker	**Series**	**Mean**	**Var**	**Skew**	**Kurtosis**	**Mean**	**Var**	**Skew**	**Kurtosis**
GSPC	S&P500 Index	0.00047	0.00006	-0.259	4.888				
AAPL	Apple Inc.	0.00058	0.00029	-0.395	9.045	0.00040	0.00021	-0.162	3.999
TMK	Torchmark Corp.	-0.00098	0.00282	-30.089	937.475	0.00057	0.00008	-0.292	4.495
BAC	Bank of America Corp	0.00106	0.00032	0.184	4.692	0.00093	0.00023	0.181	3.964
C	Citigroup Inc.	0.00060	0.00028	-0.133	4.678	0.00070	0.00023	0.017	3.910
SRCL	Stericycle Inc	0.00044	0.00014	-5.738	105.375	0.00065	0.00008	0.055	3.941
GE	General Electric	0.00053	0.00013	0.795	9.922	0.00010	0.00011	-0.082	3.996
KORS	Michael Kors Holdings	0.00039	0.00065	-0.053	27.509	0.00031	0.00036	-0.099	4.163
JPM	JPMorgan Chase & Co.	0.00063	0.00020	-0.329	6.691	0.00057	0.00016	-0.192	3.807
PFE	Pfizer Inc.	0.00038	0.00011	-0.001	4.621	0.00036	0.00010	-0.024	4.387
STZ	Constellation Brands	0.00192	0.00040	4.357	89.620	0.00129	0.00015	0.129	3.616

Table 6.2: Sample co-moments in total series

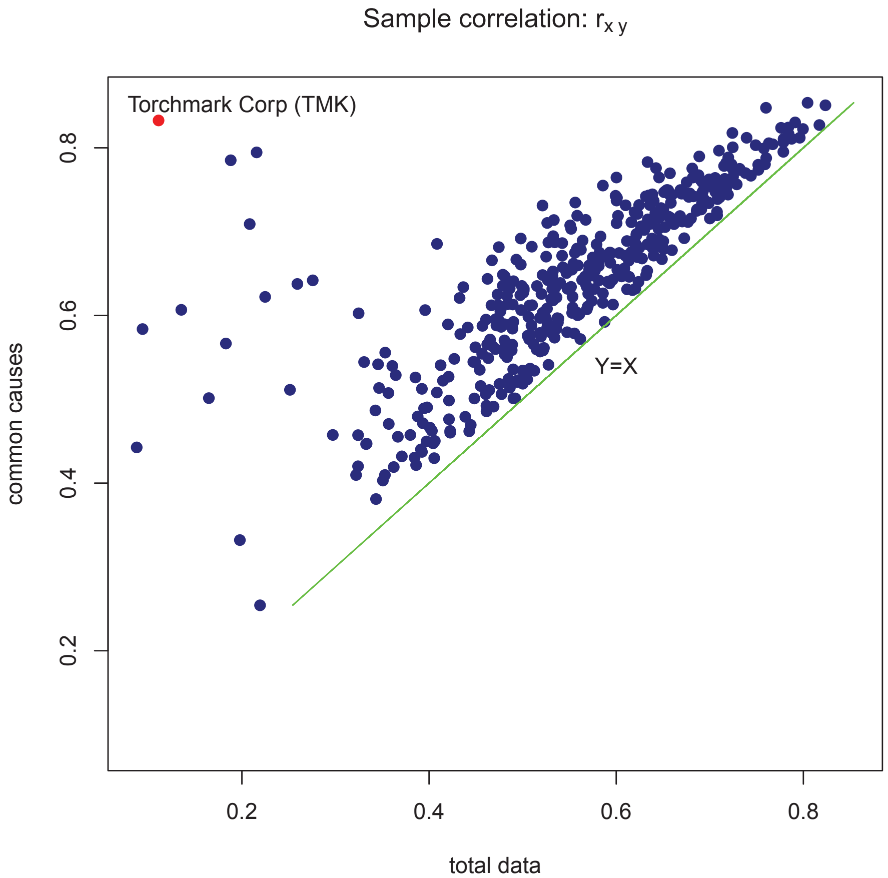
Ticker	Series	$s(x,y)$	$cs(x^2,y)$	$cs(x,y^2)$	$ck(x^3,y)$	$ck(x,y^3)$
AAPL	Apple Inc.	0.4828	0.0126	-0.0883	1.8506	2.5641
TMK	Torchmark Corp.	0.1108	0.7860	-0.0425	-24.4135	0.6638
BAC	Bank of America Corp	0.6612	-0.0527	-0.1758	2.5085	3.1663
C	Citigroup Inc.	0.7281	-0.1222	-0.1918	2.7255	3.3948
SRCL	Stericycle Inc	0.4668	0.3304	-0.1653	-5.8918	2.3528
GE	General Electric	0.7168	-0.0236	-0.1694	3.0082	3.3992
KORS	Michael Kors Holdings	0.3324	0.0149	-0.1790	-0.4574	1.7105
JPM	JPMorgan Chase & Co.	0.7241	-0.1393	-0.2162	3.1875	3.5061
PFE	Pfizer Inc.	0.6222	0.0024	-0.0962	2.6103	3.1922
STZ	Constellation Brands	0.3954	0.1981	0.0236	4.4500	2.0288

Table 6.3: Sample co-moments in common cause periods

Ticker	Series	$s(x,y)$	$cs(x^2,y)$	$cs(x,y^2)$	$ck(x^3,y)$	$ck(x,y^3)$
AAPL	Apple Inc.	0.5498	-0.1242	-0.1767	2.3571	2.8948
TMK	Torchmark Corp.	0.8319	-0.2937	-0.2894	4.0248	4.3021
BAC	Bank of America Corp	0.7083	-0.0989	-0.1658	2.7428	3.4855
C	Citigroup Inc.	0.7557	-0.1349	-0.1834	2.9884	3.5721
SRCL	Stericycle Inc.	0.6652	-0.1086	-0.2007	2.7110	3.3094
GE	General Electric	0.7814	-0.1435	-0.2036	3.2834	3.7182
KORS	Michael Kors Holdings	0.4462	-0.1282	-0.2053	1.8543	2.2947
JPM	JPMorgan Chase & Co.	0.7648	-0.2277	-0.2590	3.1226	3.7533
PFE	Pfizer Inc.	0.6689	-0.0255	-0.1031	3.0373	3.4494
STZ	Constellation Brands	0.6059	-0.0974	-0.1517	2.0989	2.8078

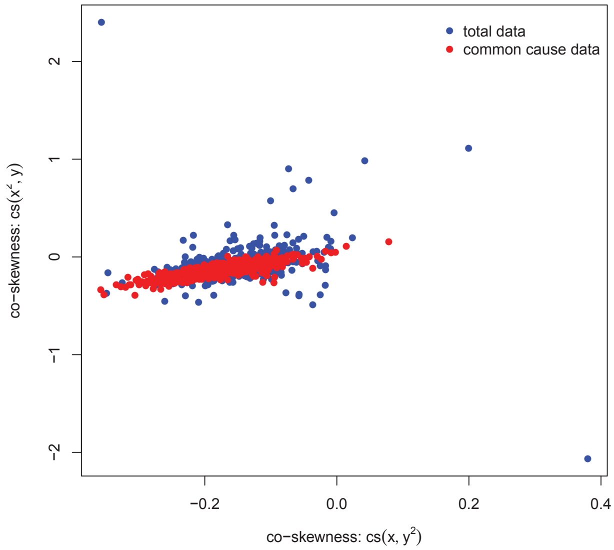
by unusual market fluctuations. Figure 6.3 is informative for investors to quantify the correlations between assets in a robust manner.

Figure 6.3: Scatter plot of sample correlation coefficient: total vs. common causes

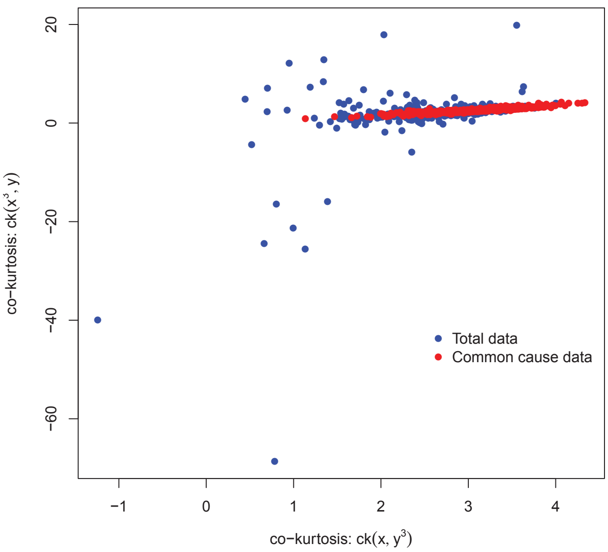


Figures 6.4a and 6.4b show the co-skewness and co-kurtosis estimates for total and common cause periods respectively. There is a greater degree of consistency in the common cause periods when compared to measures computed for the total data. This analysis provides further insight for the extended CAPM approach (Section 1.1, *Issue* 2) which models skewness and kurtosis in the return series. Investigation of the symmetry in the measures for co-skewness and co-kurtosis reveals that the common cause periods largely support the relationships pertaining to the standard bivariate normal distribution.

Figure 6.4: Co-moments in total and common cause data



(a) Co-skewness: $cs(x, y^2)$ vs. $cs(x^2, y)$



(b) Co-kurtosis: $ck(x, y^3)$ vs. $ck(x^3, y)$

6.4 Sample estimates from pairwise differences

In this section, new measures for co-moments are proposed. The sample variance can be calculated taking the sum of squares of all possible differences, see Zhang et al. (2012) for theory. Let $\left(x_i, y_i\right), i = 1, 2, ..., n$ be the pairs of the two random variables (X, Y). By considering all possible pairwise differences, the variance can be calculated as follows:

$$S_{\text{diff}} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \left(x_i - x_j\right)^2}{2n(n-1)} \tag{6.8}$$

In this section, this technique is extended to co-moments. The sample co-variance is generally calculated using equation 6.9 which considers the all 'matching' pairs of mean deviations. That is $(x_i - \bar{x}, y_i - \bar{y})$ where $i = 1, ..., n$, $\bar{x}$ and $\bar{y}$ are means of the two series X and Y respectively.

$$s(x, y) = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x}) \left(y_i - \bar{y}\right) \tag{6.9}$$

The pairwise differences can be considered in different ways, for example, the differences of matching pairs, partially matching pairs or non-matching pairs of the index. For example, Zhang et al. (2012) used all matching pairs of (X, Y) and the sample estimates for the co-variance was given as follows:

$$s(x, y)_{\text{mp}} = \frac{1}{2n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(x_i - x_j\right) \left(y_i - y_j\right) \tag{6.10}$$

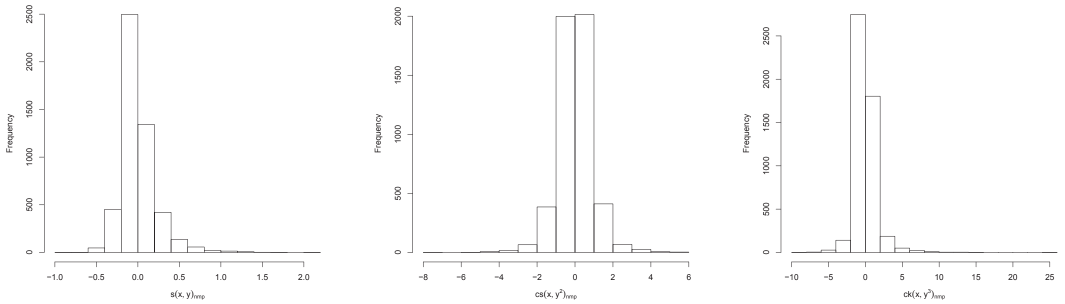
In this study, all three possible scenarios were considered but the differences based on non-matching pairs was chosen due to the computational efficiency and performance.

The formulae for the new measures of the three co-moments are given below. Note that, common to all equations given below, $i \neq j \neq k \neq l$ and $N = \frac{n(n-1)(n-2)(n-3)}{4}$.

$$
\begin{aligned}
s(x,y)_{\text{nmp}} &= \frac{1}{N}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\left(x_i - x_j\right)\left(y_k - y_l\right)\\
cs(x,y^2)_{\text{nmp}} &= \frac{1}{N}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\left(x_i - x_j\right)\left(y_k - y_l\right)^2\\
cs(x^2,y)_{\text{nmp}} &= \frac{1}{N}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\left(x_i - x_j\right)^2\left(y_k - y_l\right)\\
ck(x,y^3)_{\text{nmp}} &= \frac{1}{N}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\left(x_i - x_j\right)\left(y_k - y_l\right)^3\\
ck(x^2,y^2)_{\text{nmp}} &= \frac{1}{N}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\left(x_i - x_j\right)^2\left(y_k - y_l\right)^2\\
ck(x^3,y)_{\text{nmp}} &= \frac{1}{N}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\left(x_i - x_j\right)^3\left(y_k - y_l\right)
\end{aligned}
\tag{6.11}
$$

Theoretically the co-measures based on all differences of non-matching observations in (X,Y) series are equal to zero when the two variables are significantly correlated. The existing sample measures for co-moments retain the order among (matching pairs) the individual values of the two variables. The behaviour of the non-matching pairs is opposite to that of the matching pairs. However, Figure 6.5 shows the distribution of these sample co-moments for a sample size of 10. All three measures are centred at zero and the observed discrepancies are Monte Carlo sampling induced variation.

Figure 6.5: Distribution of sample estimates for co-variance, co-skewness and co-kurtosis based on the non-matching pairs of differences for sample size 10



(a) Co-variance **(b)** Co-skewness **(c)** Co-kurtosis

Table 6.4: False alarm rate comparison of existing co-measures and non-matching pairs based measure for co-moments

	False alarm rate					
	$n = 10$		$n = 20$		$n = 50$	
Co-moment	**Standard**	**New**	**Standard**	**New**	**Standard**	**New**
co-variance	0.143	0.108	0.167	0.103	0.149	0.119
co-skewness	0.212	0.203	0.298	0.222	0.384	0.284
co-kurtosis	0.283	0.251	0.359	0.304	0.430	0.374

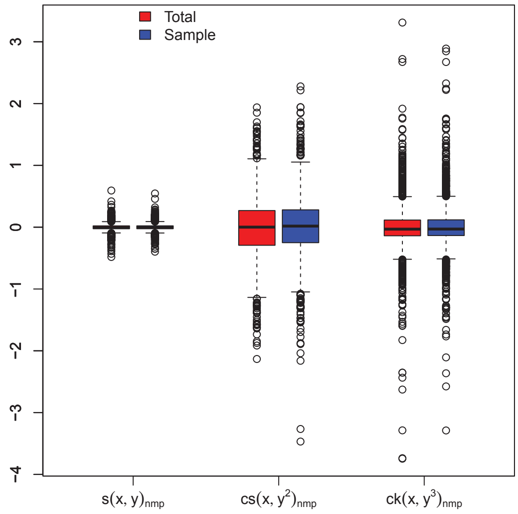
Power comparison using Monte Carlo experiment

Random data is generated from the standard bi-variate normal distribution to investigate the performance of the proposed estimates for co-moments. Empirical distributions for each co-moment estimate were obtained using 5000 replicates. Different sample sizes (n) such as 10, 20 and 50 were also opted. The upper and lower quantiles were obtained for each co-moments (equation 6.1 and 6.11).

The Y series was then contaminated by 10%. The rates of detecting the outliers are presented in Table 6.4. The new measures show approximately equal false alarm rates to the standard measures (equation 6.1).

When the sample size increases, the number of all possible non-matching pairs becomes very large. It is not computationally efficient to consider all these pairs to calculate

Figure 6.6: $s(x, y)_{nmp}$, $cs(x, y^2)_{nmp}$ and $ck(x, y^3)_{nmp}$ for the total series and a sub-sample size 20

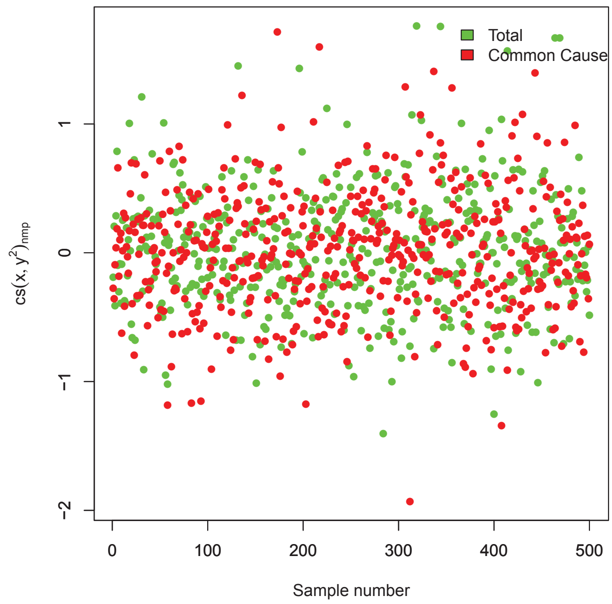


these measures; therefore, these estimates are reinvestigated using a small sub-sample. Figure 6.6 shows the box plots for the three measures using all possible pairs and a sub-sample. From the total series of 40 observations, a random sub-sample of 20 observations was chosen without replacement. Results based on the sub-sample do not show a significant deviation from the total data. It is therefore justified to use a sample of all possible pairs to maintain the computational efficiency.

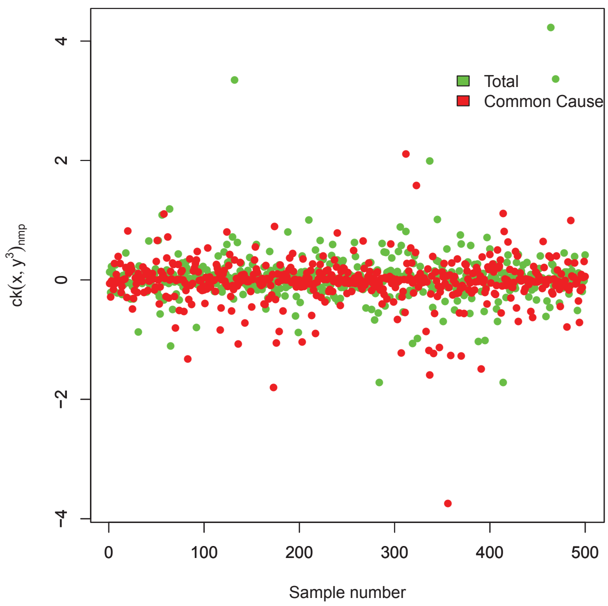
The behaviour of these new measures for empirical data is demonstrated using the following example. New estimates for co-moments based on non-matching pairs were also investigated in partitioned data. A stock, Michael Kors Holdings (KORS) was chosen from Table 6.1. The number of observations in this series is about 1000, and hence a random sample of 30 observations was selected to obtain all the possible non-matching pairs. When the total data is contaminated, one sub-sample may not be sufficient. Several sub-samples from the data series are considered to calculate co-moments in both total and common cause periods. Figures 6.7a and 6.7b clearly show unusual values in the total data as expected.

Whilst beyond the scope of this current study, these tentative results suggest that statistical significance tests for excess co-moments based on the newly proposed measures are promising. A further investigation is theretofore proposed.

Figure 6.7: Newly proposed co-moments measures in total and common cause data using stock returns of Michael Kors Holdings (KORS)



(a) $cs(x, y^2)_{nmp}$



(b) $ck(x, y^3)_{nmp}$

6.5 Summary

The aim of this chapter was to examine the following proposition:

Proposition 3: The joint distribution of an asset return and the market return will follow a bivariate normal distribution under common cause variations.

The importance of co-moments in stock returns were investigated in the partitioned data for validating Shewhart postulates for finance data. For the standard bivariate normal distribution, symmetric behaviour was found for standard measures for co-skewness and co-kurtosis. These results were used to examine effect of special causes. Empirical verification of symmetry in the co-moments for total and common cause periods was presented as the justification for *Proposition* 2. The use of Shewhart methods for partitioning the data helps to understand the role of co-moments and their possible use on asset pricing models in a future study. The analysis presented in this chapter emphasised the need for new formulas/measures of co-skewness and co-kurtosis when total data is used for the extended CAPM analysis. A new set of estimators which is based on the differences of non-matching pairs of observations in the two variables was proposed. Simulation results, even though limited, showed that these measures performed equally well compared to the existing measures.

Chapters 7 and 8 divert the study to examine the autocorrelation in stock returns which was identified in Section 1.1 as an another central issue needing to be investigated in a Shewhart's point of view.

Chapter 7

Odd-even split

Autocorrelation in stock returns is one of the most contentious issue for financial researchers. When applying the Shewhart principles for time series data, autocorrelation observed in the return series cannot be ignored. It is, however, due to the effect of various types of special cause events in the market, the true autocorrelation in stock returns may not be recognized.

Autocorrelation in the complete (total) series was examined by many. Empirical findings for the presence of autocorrelation in stock returns reported in the literature is not consistent. The true autocorrelation has variously been claimed to be not significant, positive, negative or spurious. For this study, understanding the true nature of autocorrelation in stock returns is important.

Two non-overlapping series can be obtained from the complete series of returns according to whether the discrete time index of each observation is odd or even. The partitioning of a complete time series into two segments based on the odd and even indices is used for some applications (e.g. speech recognition) but rarely found in finance applications. The aim of splitting the complete series is twofold: to avoid the autocorrelation effect of having the same price terms in consecutive stock returns and to exploit the fact that autocorrelation must be equal in odd and even series.

The theoretical autocorrelation and partial autocorrelation functions of the partitioned odd-even series for autoregressive moving average models are not explicitly stated in the literature. This chapter provides the autocorrelation properties of odd-even series for known linear time series models and compares them with the complete series.

7.1 Introduction

Segmenting a time series into parts is not totally new. Time series segmentation is used in data mining and speech recognition applications, see Fu (2011); Appel and Brandt (1983); Kehagias and Petridis (1997); Himberg et al. (2001). In Wavelet analysis, odd-even splitting is usually considered as a natural way of filtering data (see Narang et al., 2010; Kingsbury, 2001). Using the odd and even indices of a complete time series, two disjoint sub-series can be formed. This partitioning procedure will be referred to as the odd-even split in this study. The *odd-even split* is a fixed pattern of partitioning. The odd-even series has been employed by Jayant (1981) and Goyal (2001) in speech transmission applications. The odd-even channel split of a single call has been used to improve the reliability of a telephone system without a standby connection. In their application, the reduced autocorrelation in the sub-series was considered in order to increase the performance in decoding.

Theoretical properties of systematic sampling and aggregation methods for financial data have been discussed by many authors, including Brewer (1973); Wei (1981); Granger and Siklos (1995); Hassler (2011). However, Rossana and Seater (1995) has been shown the information loss caused by the aggregation because this method hides the low-frequency variation. The available sampled time series of data is partitioned into odd and even series of data. This approach of odd-even partitioning differs from the systematic sampling of an underlying process discussed by Brewer (1973) and others. Even though the odd-even split is equivalent to systematic sampling of the sample data with a frequency of 1/2, the discussion in Brewer (1973) and others related to the loss of information due to the sampling frequency. The odd-even partitioning technique newly introduced in this chapter does not result in loss of information because the complete as well as both the partitioned series are used in the analysis. In the systematic sampling approach, the higher the sampling frequency, the smaller the loss of information. The odd-even split approach discussed in this chapter deals with extracting further information from a sampled time series. Some of the results derived for k^{th}-skip sampling for ARMA models by Brewer (1973) are equivalent to the odd-even split ($k = 2$) proposed in this chapter. However the essential difference is that our proposal makes use of both $k = 1$ (original sampled series) and $k = 2$ (odd-even split series) data for analysis.

In the control chart literature, a data skipping strategy was considered in Costa and Castagliola (2011); Franco et al. (2014) so that the chart implementation becomes simpler due to the dampening of the autocorrelation effect. The odd-even split strategy is expected to be more powerful than the data skipping procedure because the autocorrelation effect on the original series can be partly recovered from the two partitioned series. For example, any inconsistency in the control chart signals generated from the odd-even series may be an indication of special causes.

The autocorrelation properties of the odd-even partitioned series of a known linear time series model have not been documented in the literature. In this chapter, the autocorrelation functions (ACFs) and partial autocorrelation functions (PACFs) of the odd-even series for autoregressive moving average (ARMA) models are studied.

The remainder of this chapter is organized as follows. The ACF/PACFs for odd-even series of linear time series models are derived in Section 7.2. Sample ACFs for the complete series can be recombined using odd and even series, and the relationship between ACFs of complete series and partitioned series is explained in Section 7.3. Section 7.4 summarises the chapter.

7.2 Partitioning via the Odd-even split

Let us denote the complete discrete time series as $\{x_t;\ t = 1, ..., n\}$, and for simplicity, let n be even. The complete series can be split into an odd series $x_{t,\mathrm{o}} : \{x_{2t-1};\ t = 1, \ldots, \frac{n}{2}\}$ and an even series $x_{t,\mathrm{e}} : \{x_{2t};\ t = 1, \ldots, \frac{n}{2}\}$ based on the odd and even time indices. The theoretical ACF/PACFs of the odd-even series for certain common linear time series models are derived below. The model specifications, notations and definitions are listed (largely quoted from Duncan, 1979; Box et al., 2011) in Table 7.1.

Table 7.1: Model specifications, notations and definitions for autocorrelation analysis

Symbols	**Description**
$\{x_t;\ t = 1, ..., n\}$	a complete series
$x_{t,\text{o}} : \{x_{2t-1};\ t = 1, \ldots, \frac{n}{2}\}$	odd series
$x_{t,\text{e}} : \{x_{2t};\ t = 1, \ldots, \frac{n}{2}\}$	even series
$\gamma_k = \text{Cov}[x_t, x_{t-k}]$	autocovariance at lag k of the complete series
$\gamma_{k,\text{o}}$ and $\gamma_{k,\text{e}}$	autocovariance at lag k of the odd and even series respectively
$\rho_k = \frac{\gamma_k}{\gamma_0}$	autocorrelation at lag k of the complete series
$\rho_{k,\text{o}}$ and $\rho_{k,\text{e}}$	autocorrelation at lag k of the odd and even series respectively
r_k	sample autocorrelation function at lag k of the complete series
$r_{k,\text{o}}$ and $r_{k,\text{e}}$	sample autocorrelation at lag k of the odd and even series respectively
$\text{Corr}(x_{t,\text{o}}[t], x_{t,\text{e}}[t])$	two-sided zero lag cross-correlation between odd and even series
$\text{Corr}(x_{t,\text{o}}[t+k], x_{t,\text{e}}[t])$	two-sided cross-correlation between odd and even series at lag k
ϕ_{kk}	partial autocorrelation function of the complete series
$\phi_{kk,\text{o}}$ and $\phi_{kk,\text{e}}$	partial autocorrelation at lag k of the odd and even series respectively

r_{kk}	sample partial autocorrelation function of the complete series
$r_{kk,\mathrm{o}}$ and $r_{kk,\mathrm{e}}$	sample partial autocorrelation at lag k of the odd and even series respectively
$\mathrm{E}[\epsilon_t] = 0, \mathrm{Var}[\epsilon_t] = \sigma_\epsilon^2,$	white noise (WN) series
$x_t = \mu + \sum_{i=1}^{p} \phi_i x_{t-i} + \epsilon_t$	AR(p): autoregressive model of order p
$x_t = \mu + \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j}$	MA(q): moving average model of order q
$x_t = \mu + \sum_{i=1}^{p} \phi_i x_{t-i} + \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j}$	ARMA(p,q): autoregressive moving average model

7.2.1 Odd-even split of the AR(p) model

This section derives the autocorrelation functions in the odd-even split of an autoregressive model of order p. Consider the AR(1) model for the complete series and its four consecutive terms (starting with an even t):

$$x_{t-3} = \phi_1 x_{t-4} + \epsilon_{t-3}$$

$$x_{t-2} = \phi_1 x_{t-3} + \epsilon_{t-2}$$

$$x_{t-1} = \phi_1 x_{t-2} + \epsilon_{t-1}$$

$$x_t = \phi_1 x_{t-1} + \epsilon_t$$

where x_{t-2} and x_t are the $\left(\frac{t}{2}-1\right)^{\text{th}}$ and $\left(\frac{t}{2}\right)^{\text{th}}$ terms of the even series respectively. After substitution and simplification,

$$x_{t-2} = \phi_1^2 x_{t-4} + \phi_1 \epsilon_{t-3} + \epsilon_{t-2}$$

$$x_t = \phi_1^2 x_{t-2} + \phi_1 \epsilon_{t-1} + \epsilon_t$$

Similarly, by considering the four consecutive terms, x_{t-2}, x_{t-1}, x_t and x_{t+1}, the odd series exhibits the following similar dependency structure as in the even series:

$$x_{t-1} = \phi_1^2 x_{t-3} + \phi_1 \epsilon_{t-2} + \epsilon_{t-1}$$
$$x_{t+1} = \phi_1^2 x_{t-1} + \phi_1 \epsilon_t + \epsilon_{t+1}$$

The even series part for the AR(1) model becomes

$$x_{t,\mathrm{e}} = \phi_1^2 x_{t-1,\mathrm{e}} + \epsilon_t^*$$

where $\epsilon_t^* = \phi_1 \epsilon_{t-1} + \epsilon_t$, $\epsilon_{t-1}^* = \phi_1 \epsilon_{t-3} + \epsilon_{t-2}$, $\mathrm{E}[\epsilon_t^*] = 0$, $\mathrm{Var}[\epsilon_t^*] = (\phi_1^2 + 1)\sigma^2$, $\mathrm{Cov}[\epsilon_{t-1}^* \epsilon_t^*] = 0$. Since the ϵ_t^*s are also independent, the even (or odd) series of AR(1) model is equivalent to the AR(1) model with a new coefficient ϕ_1^2. The even series of the AR(1) model is stationary because $-1 < \phi_1^2 < 1$. The ACFs/PACFs of the even series of AR(1) model can be derived in a similar fashion to the case of a complete AR(1) model. The ACF at lag k of the even series is

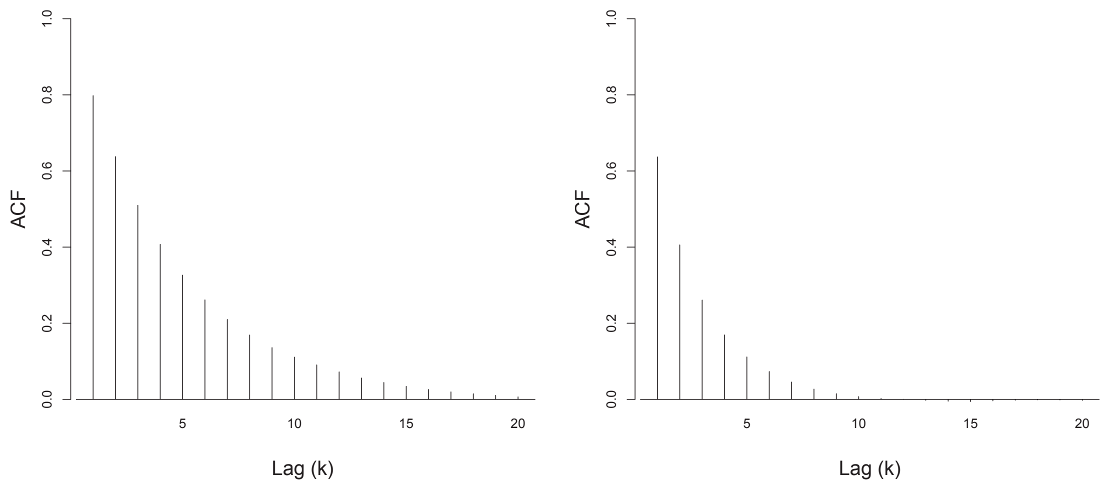
$$\rho_{k,\mathrm{e}} = \phi_1^{2k}$$

The PACF function of AR(1) model cuts off at lag 1, $\phi_{11,\mathrm{e}} = \rho_{1,\mathrm{e}} = \phi_1^2$. Figure 7.1 gives the sample ACFs of an AR(1) model and its even series. The sample PACFs of the even series cuts off at lag one and $r_{11,\mathrm{e}} \approx \phi_1^2$.

For the AR(2) model $x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \epsilon_t$, the adjacent terms of the even series can be written in the form

$$x_{t-2} = (\phi_1^2 + 2\phi_2) x_{t-4} - \phi_2^2 x_{t-6} - \phi_2 \epsilon_{t-4} + \phi_1 \epsilon_{t-3} + \epsilon_{t-2}$$
$$x_t = (\phi_1^2 + 2\phi_2) x_{t-2} - \phi_2^2 x_{t-4} - \phi_2 \epsilon_{t-2} + \phi_1 \epsilon_{t-1} + \epsilon_t$$

Figure 7.1: Sample ACFs for AR(1) model: $x_t = 0.8x_{t-1} + \epsilon_t$ and its even series $x_{t,\mathrm{e}}$: $x_t = 0.64x_{t-2} + \epsilon_t^*$



(a) ACFs of AR(1): $x_t = 0.8x_{t-1} + \epsilon_t$ model

(b) ACFs of the even series: $x_{t,\mathrm{e}}$: $x_t = 0.64x_{t-2} + \epsilon_t^*$, where $\epsilon_t^* = 0.8\epsilon_{t-1} + \epsilon_t$

This gives the following general form for the even series for the AR(2) model

$$x_{t,\mathrm{e}} = (\phi_1^2 + 2\phi_2)x_{t-1,\mathrm{e}} - \phi_2^2 x_{t-2,\mathrm{e}} + \epsilon_t^*,$$

$$\text{where} \quad \epsilon_t^* = -\phi_2\epsilon_{t-2} + \phi_1\epsilon_{t-1} + \epsilon_t$$

$$\epsilon_{t-1}^* = -\phi_2\epsilon_{t-4} + \phi_1\epsilon_{t-3} + \epsilon_{t-2}$$

The error terms show lag one dependency and hence the even series of an AR(2) model becomes an ARMA(2,1) model. To obtain the autocorrelation function, the autocovariance function is first simplified and then divided by the variance.

$$\begin{aligned}\gamma_{k,\mathrm{e}} &= \mathrm{E}[x_{t,\mathrm{e}}x_{t-k,\mathrm{e}}] \\ &= (\phi_1^2 + 2\phi_2)\mathrm{E}[x_{t-1,\mathrm{e}}x_{t-k,\mathrm{e}}] - \phi_2^2\mathrm{E}[x_{t-2,\mathrm{e}}x_{t-k,\mathrm{e}}] + \mathrm{E}[\epsilon_t^* x_{t-k,\mathrm{e}}]\end{aligned}$$

The autocovariance function for all $k = 0, 1, 2, \ldots$ can be found recursively.

$$\gamma_{0,\mathrm{e}} = (\phi_1^2 + 2\phi_2)\gamma_{1,\mathrm{e}} - \phi_2^2\gamma_{2,\mathrm{e}} + \mathrm{E}[\epsilon_t^* x_{t,\mathrm{e}}]$$

Consider

$$\begin{aligned}
E[\epsilon_t^* x_{t,\mathrm{e}}] &= \mathrm{E}[\epsilon_t^* \left((\phi_1^2 + 2\phi_2)x_{t-1,\mathrm{e}} - \phi_2^2 x_{t-2,\mathrm{e}} + \epsilon_t^*\right)] \\
&= (\phi_1^2 + 2\phi_2)\mathrm{E}[x_{t-1,\mathrm{e}}\epsilon_t^*] - \phi_2^2\mathrm{E}[x_{t-2,\mathrm{e}}\epsilon_t^*] + \mathrm{E}[\epsilon_t^*\epsilon_t^*] \\
&= \left(1 + \phi_1^2 - \phi_2^2 - \phi_2\phi_1^2\right)\sigma^2 \\
&= (\phi_1^2 + 2\phi_2)\gamma_{1,\mathrm{e}} - \phi_2^2\gamma_{2,\mathrm{e}} + \left(1 + \phi_1^2 - \phi_2^2 - \phi_2\phi_1^2\right)\sigma^2
\end{aligned}$$

Similarly

$$\begin{aligned}
\gamma_{1,\mathrm{e}} &= (\phi_1^2 + 2\phi_2)\gamma_{0,\mathrm{e}} - \phi_2^2\gamma_{1,\mathrm{e}} + \mathrm{E}[\epsilon_t^* x_{t-1,\mathrm{e}}] \\
E[\epsilon_t^* x_{t-1,\mathrm{e}}] &= \mathrm{E}[\epsilon_t^* \left((\phi_1^2 + 2\phi_2)x_{t-2,\mathrm{e}} - \phi_2^2 x_{t-3,\mathrm{e}} + \epsilon_{t-1}^*\right)] = -\phi_2\sigma^2 \\
\gamma_{1,\mathrm{e}} &= (\phi_1^2 + 2\phi_2)\gamma_{0,\mathrm{e}} - \phi_2^2\gamma_{1,\mathrm{e}} - \phi_2\sigma^2 \\
\gamma_{2,\mathrm{e}} &= (\phi_1^2 + 2\phi_2)\gamma_{1,\mathrm{e}} - \phi_2^2\gamma_{0,\mathrm{e}} \\
\gamma_{k,\mathrm{e}} &= (\phi_1^2 + 2\phi_2)\gamma_{k-1,\mathrm{e}} - \phi_2^2\gamma_{k-2,\mathrm{e}} \quad \text{for} \quad k > 2
\end{aligned}$$

Using the expressions of $\gamma_{0,\mathrm{e}}, \gamma_{1,\mathrm{e}}$ and $\gamma_{2,\mathrm{e}}$, the autocovariances can be found as functions of ϕ_1, θ_1 and σ^2. The autocorrelation function can be obtained by dividing by $\gamma_{0,\mathrm{e}}$. Similarly, the ACFs of the odd series of an AR(2) model can be found.

For the even series of the AR(p) model ($p > 2$), replace all odd indexed terms using even indexed terms is needed. This is done backwards, starting from x_{t-1} where

$$x_{t-1} = \phi_1 x_{t-2} + \phi_2 x_{t-3} + \ldots + \phi_p x_{t-1-p} + \epsilon_{t-1}$$

Then,

$$x_t = \phi_1 \left(\phi_1 x_{t-2} + \phi_2 x_{t-3} + ... + \phi_p x_{t-1-p} + \epsilon_{t-1}\right) + \phi_2 x_{t-2} + ... + \phi_p x_{t-p} + \epsilon_t$$

$$x_t = \phi_1^2 x_{t-2} + \phi_1 \phi_2 x_{t-3} + \phi_1 \left(\phi_3 x_{t-4} + ... + \phi_p x_{t-1-p} + \epsilon_{t-1}\right) + \phi_2 x_{t-2} + ... + \phi_p x_{t-p} + \epsilon_t$$

Consider,

$$x_{t-2} = \phi_1 x_{t-3} + \phi_2 x_{t-4} + ... + \phi_p x_{t-2-p} + \epsilon_{t-2}$$

$$\phi_2 x_{t-2} = \phi_2 \left(\phi_1 x_{t-3} + \phi_2 x_{t-4} + ... + \phi_p x_{t-2-p} + \epsilon_{t-2}\right)$$

Then,

$$\begin{aligned} x_t &= \phi_1^2 x_{t-2} + \left(\phi_2 x_{t-2} - \phi_2 \left(\phi_2 x_{t-4} + ... + \phi_p x_{t-2-p} + \epsilon_{t-2}\right)\right) \\ &+ \phi_1 \left(\phi_3 x_{t-4} + ... + \phi_p x_{t-1-p} + \epsilon_{t-1}\right) + \phi_2 x_{t-2} + ... + \phi_p x_{t-p} + \epsilon_t \\ x_t &= \left(\phi_1^2 + 2\phi_2\right) x_{t-2} + \left(-\phi_2 \left(\phi_2 x_{t-4} + ... + \phi_p x_{t-2-p} + \epsilon_{t-2}\right)\right) \\ &+ \phi_1 \left(\phi_3 x_{t-4} + ... + \phi_p x_{t-1-p} + \epsilon_{t-1}\right) + ... + \phi_p x_{t-p} + \epsilon_t \end{aligned}$$

The substitution is carried out up to the term x_{t-2p}. Note that for odd series, the even indexed terms are substituted by the odd indexed terms. The model can then be written as

$$\begin{aligned} x_{t,\mathrm{e}} &= (\phi_1^2 + 2\phi_2) x_{t-1,\mathrm{e}} + ... + (...) x_{t-p,\mathrm{e}} + \epsilon_t^*, \\ \text{where} \quad \epsilon_t^* &= (...)\epsilon_{t-2p} + ... + \phi_1 \epsilon_{t-1} + \epsilon_t \end{aligned}$$

For the even series, the ϵ_t^*'s show lag $\frac{p}{2}$ and $\frac{p-1}{2}$ dependency for even and odd values of order p respectively. The odd series can be handled similarly. The even-odd series of AR(p) is therefore equivalent to ARMA(p, $\frac{p}{2}$) and ARMA(p, $\frac{p-1}{2}$) when p is even and odd respectively. All the model coefficients are functions of ϕ_i. A similar derivation is reported in Brewer (1973, p.141) where the order of the k^{th} skip sampling of an ARMA process leads to $\text{ARMA}\big(p, (p(k-1)+q)/k\big)$ when $k=2$ and $q=0$.

7.2.2 Odd-even split of the MA(q) model

The even series of moving average model of order q models are identifiable with their AFC/PACFs. Consider the MA(1) model. Take the four adjacent terms, (x_{t-2}, x_{t-1}, x_t, and x_{t+1}) of this MA(1) model,

$$x_{t-2} = \epsilon_{t-2} + \theta_1 \epsilon_{t-3}$$

$$x_{t-1} = \epsilon_{t-1} + \theta_1 \epsilon_{t-2}$$

$$x_t = \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$x_{t+1} = \epsilon_{t+1} + \theta_1 \epsilon_t$$

For even t, $\gamma_{1,\text{e}}$ is given by

$$\gamma_{1,\text{e}} = \text{E}\left[(\epsilon_t + \theta_1 \epsilon_{t-1})\,(\epsilon_{t-2} + \theta_1 \epsilon_{t-3})\right] = 0$$

For the odd series of the MA(1) model,

$$\gamma_{1,\text{o}} = \text{E}\left[(\epsilon_{t-1} + \theta_1 \epsilon_{t-2})\,(\epsilon_{t+1} + \theta_1 \epsilon_{t-1})\right] = 0$$

The covariance between the consecutive terms is zero, and the variance of the even or odd series is equal to $(1+\theta_1^2)\sigma^2$. Hence the odd-and even series of MA(1) model are equivalent to the white noise (WN) model.

Now, consider the odd-even series of the MA(2) model

$$\gamma_{1,\mathrm{e}} = \mathrm{E}\left[\left(\epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2}\right)\left(\epsilon_{t-2} + \theta_1\epsilon_{t-3} + \theta_2\epsilon_{t-4}\right)\right] = \theta_2$$

$$\gamma_{2,\mathrm{e}} = 0, k > 1$$

The ACF of the even series cuts off after lag 1 as does the odd series. The odd-even series of MA(2) model shows lag one dependence and is therefore equivalent to the MA(1) model; and,

$$\rho_{1,\mathrm{e}} = \frac{\theta_2}{1+\theta_1^2+\theta_2^2}$$

Similarly, it can be showed that the ACF of the odd-even series of MA(3) model also cuts off after lag 1. The ACF at lag 1 of the odd-even series of the MA(3) model

$$\rho_{1,\mathrm{e}} = \frac{\theta_2+\theta_1\theta_3}{1+\theta_1^2+\theta_2^2+\theta_3^2}$$

The odd-even series of MA(2) or MA(3) models are equivalent to a MA(1) model. Figure 7.2 shows the sample ACFs of a MA(3) model, and its even series and $r_{1,\mathrm{e}} \approx \frac{\theta_2+\theta_1\theta_3}{1+\theta_1^2+\theta_2^2+\theta_3^2}$.

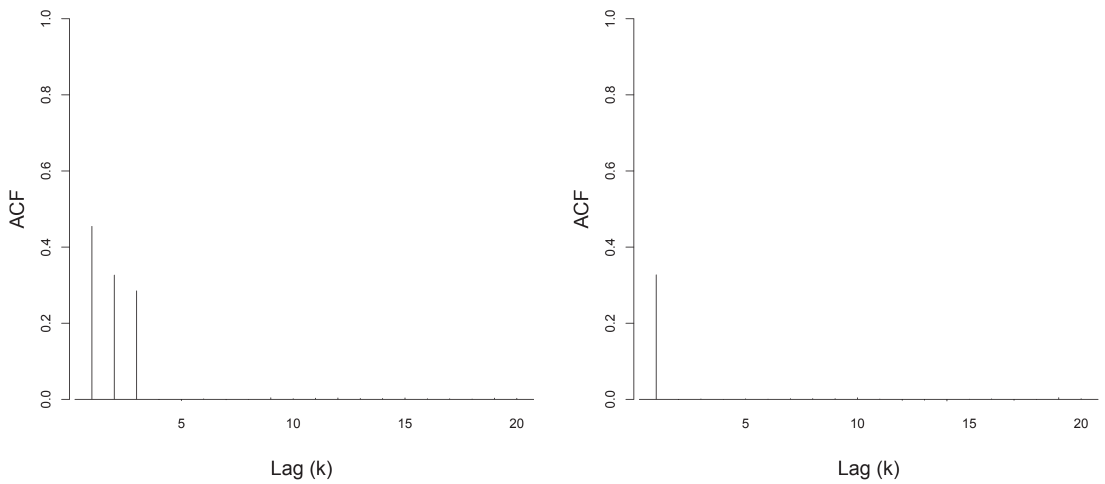
For the general MA(q) model (q even), the odd-even series can be extracted as follows. Consider

$$x_t = \epsilon_t + \theta_1\epsilon_{t_1} + \theta_2\epsilon_{t_2} + \theta_3\epsilon_{t_3} + \ldots + \theta_q\epsilon_{t_q}$$

and the $(t+q)^{\mathrm{th}}$ term:

$$x_{t+q} = \epsilon_t + \theta_1\epsilon_{t_1} + \theta_2\epsilon_{t_2} + \theta_3\epsilon_{t_3} + \ldots + \theta_q\epsilon_{t_q}$$

Figure 7.2: Sample ACFs for MA(3) model : $x_t = \epsilon_t + 0.4\epsilon_{t-1} + 0.3\epsilon_{t-2} + \epsilon_{t-3}$ and its even series



(a) ACFs of MA(3) model **(b)** ACFs of the even series of MA(3) model

are the $\frac{t}{2}^{\text{th}}$ and $\frac{(t+q)}{2}^{\text{th}}$ terms of the even series of the MA(q) model respectively (assuming that t is even). The covariance functions are non-zero until lag $k = \frac{q}{2}$ and

$$\gamma_{\frac{q}{2},\text{e}} = \theta_q$$

The even (odd) series becomes a MA($\frac{q}{2}$) model. The PACFs have not been discussed in detail because the odd-even series of the MA models also show a structure similar to the corresponding complete series. As a consequence, the duality between AR and MA models can be used to derive the PACFs. The odd-even series of MA($q+1$) model also gives the ACF/PACF behaviour of MA($\frac{q}{2}$) model.

7.2.3 Odd-even split of the ARMA(p, q) model

Autocorrelation functions for the odd-even series of autoregressive moving average model are derived in this section. Consider the ARMA(1,1) model

$$x_{t-2} = \phi_1 x_{t-3} + \epsilon_{t-2} + \theta_1 \epsilon_{t-3}$$

$$x_{t-1} = \phi_1 x_{t-2} + \epsilon_{t-1} + \theta_1 \epsilon_{t-2}$$

$$x_t = \phi_1 x_{t-1} + \epsilon_t + \theta_1 \epsilon_{t-1}$$

$$x_{t+1} = \phi_1 x_t + \epsilon_{t+1} + \theta_1 \epsilon_t$$

and x_{t-1} and x_{t-2} terms can be used to derive the even series as follows:

$$x_t = \phi_1^2 x_{t-2} + \epsilon_t + (\phi_1 - \theta_1)\epsilon_{t-1} + \phi_1\theta_1\epsilon_{t-2}$$

$$x_{t,\mathrm{e}} = \phi_1^2 x_{t-1,\mathrm{e}} + \epsilon_t^*, \quad \text{where} \quad \epsilon_t^* = \epsilon_t + (\phi_1 - \theta_1)\epsilon_{t-1} + \phi_1\theta_1\epsilon_{t-2}$$

and for the odd series

$$x_{t+1} = \phi_1^2 x_{t-1} + \epsilon_{t+1} + (\phi_1 - \theta_1)\epsilon_t + \phi_1\theta_1\epsilon_{t-1}$$

$$x_{t,\mathrm{o}} = \phi_1^2 x_{t-1,\mathrm{o}} + \epsilon_t^*, \quad \text{where} \quad \epsilon_t^* = \epsilon_t + (\phi_1 - \theta_1)\epsilon_{t-1} + \phi_1\theta_1\epsilon_t$$

The odd and even series of the ARMA(1,1) model is equivalent to ARMA(1,1) because ϵ_t^*'s show lag 1 dependency. Similarly, the odd-even series of the ARMA(p,q) model can be derived as follows, assume that p is even:

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \ldots + \phi_p x_{t-p} + \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j}$$

Replace the x_{t-1} using

$$
\begin{aligned}
x_{t-1} &= \phi_1 x_{t-2} + \phi_2 x_{t-3} + \ldots + \phi_p x_{t-1-p} + \epsilon_{t-1} + \sum_{j=1}^{q} \theta_j \epsilon_{t-1-j} \\
x_t &= \phi_1 \left(\phi_1 x_{t-2} + \phi_2 x_{t-3} + \ldots + \phi_p x_{t-1-p} + \epsilon_{t-1} + \sum_{j=1}^{q} \theta_j \epsilon_{t-1-j} \right) \\
&\quad + \phi_2 x_{t-2} + \ldots + \phi_p x_{t-p} + \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} \\
x_t &= \phi_1^2 x_{t-2} + \phi_1 \phi_2 x_{t-3} + \phi_1 \left(\phi_3 x_{t-4} + \ldots + \phi_p x_{t-1-p} + \epsilon_{t-1} + \sum_{j=1}^{q} \theta_j \epsilon_{t-1-j} \right) \\
&\quad + \phi_2 x_{t-2} + \phi_3 x_{t-3} + .. + \phi_p x_{t-p} + \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j}
\end{aligned}
$$

Then substitute $\phi_1 \phi_2 x_{t-3}$ and $\phi_3 x_{t-3}$ from following two equations:

$$
\begin{aligned}
x_{t-2} &= \phi_1 x_{t-3} + \phi_2 x_{t-4} + \ldots + \phi_p x_{t-2-p} + \epsilon_{t-2} + \sum_{j=1}^{q} \theta_j \epsilon_{t-2-j} \\
x_{t-3} &= \phi_1 x_{t-4} + \phi_2 x_{t-5} + \ldots + \phi_p x_{t-3-p} + \epsilon_{t-3} + \sum_{j=1}^{q} \theta_j \epsilon_{t-3-j}
\end{aligned}
$$

Similarly all the odd terms of the x_t expression can be replaced until x_{t-p} where,

$$
x_{t-p} = \phi_1 x_{t-p-1} + \phi_2 x_{t-p-2} + \ldots + \phi_p x_{t-2p} + \epsilon_{t-p} + \sum_{j=1}^{q} \theta_j \epsilon_{t-p-j}
$$

Since x_t can be expressed in terms of $x_{t-2}, x_{t-4}, \ldots, x_{t-2p}$, the even series of ARMA(p,q) model shows order p dependence between the AR components. The error term of the even series becomes,

$$
\epsilon_t^* = \epsilon_t + (\ldots) \sum_{j=1}^{q} \theta_j \epsilon_{t-1-j} + (\ldots) \epsilon_{t-p} + \sum_{j=1}^{q-1} (\ldots) \epsilon_{t-p-j} + (\ldots) \epsilon_{t-(p+q)}
$$

Hence the order of the MA components becomes $\left(\frac{p+q}{2}\right)$ for even $\left(p+q\right)$ and $\left(\frac{p+q-1}{2}\right)$ for odd $\left(p+q\right)$. The corresponding ACF/PACF of ARMA(p,q) models can be derived using the approach suggested in Box et al. (2011, pp.79-82).

In summary, the odd-even split of a WN series results in WN split series; for AR(p), odd-even series become ARMA(p, $\frac{p}{2}$) when p is even and ARMA(p, $\frac{p-1}{2}$) when p is odd; for MA(1) model, odd-even series is WN and MA(q) or MA($q+1$) results the odd-even series as MA($\frac{q}{2}$), where q is even; for ARMA(p,q) model, the even(odd) series is equivalent to ARMA(p,$\left(\frac{p+q}{2}\right)$), when $p+q$ is even and ARMA(p,$\left(\frac{p+q-1}{2}\right)$) when $p+q$ is odd.

Non-linear time series models are often proposed to capture non-constant variance. The ARCH (autoregressive conditional heteroskedasticity) model proposed by Engle (1982) and GARCH (generalized ARCH) proposed by Bollerslev (1986) are considered. The odd-even split of those models does not show significant ACF/PACFs as they are not sensitive to linear ACF/PACFs by definition. A mixture of the ARCH/GARCH model for conditional variance and the ARMA specification for conditional mean was also considered as in Lange et al. (2011); Francq and Zakoïan (2004). The inclusion of a non-constant variance component in these models does not show any impact on the ACF/PACFs of the odd-even split series. The odd-even series of these models are equivalent to the corresponding odd-even series of those ARMA components.

7.3 Recombined ACFs of the complete series using odd-even split

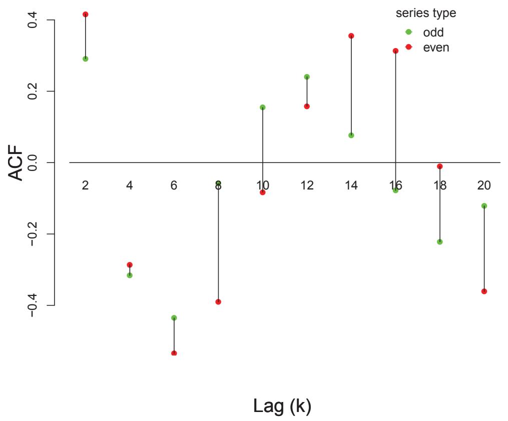
The odd-even split gives two estimates for the sample ACFs of the complete series. The sample ACFs in even lags of the complete series can be obtained from the ACFs of either odd or even series. First consider the lag 2 ACF of the complete series. This lag 2 ACF of the initial series can be calculated using the pairs: $(x_1,x_3),(x_2,x_4),(x_3,x_5),(x_4,x_6),\ldots,$ $(x_{n-3},x_{n-1}),(x_{n-2},x_n)$. The lag 1 sample ACF of the even and odd series take $(x_2,x_4),(x_4,x_6)$

, ..., (x_{n-2}, x_n) and $(x_1, x_3), (x_3, x_5), ..., (x_{n-3}, x_{n-1})$ pairs of observations respectively. These two sets of observations are subsets of the observations that are used for lag 2 of the sample ACF in the complete series, and gives two estimates.

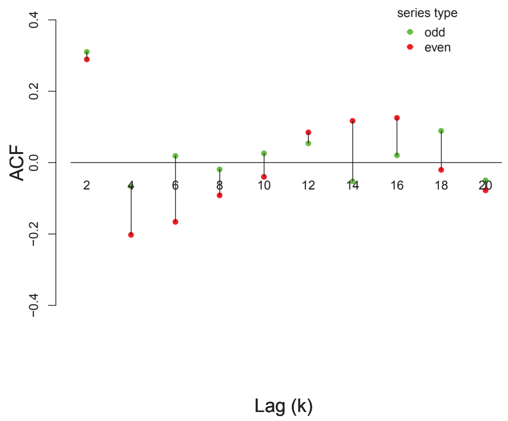
The sample ACF values in odd lags of the complete series can be computed from the cross-correlations between the odd and even series. For example, lag 1 ACF in the complete series can be estimated from both Corr($x_{t,\mathrm{o}}[t]$,$x_{t,\mathrm{e}}[t]$) and Corr($x_{t,\mathrm{o}}[t+1]$, $x_{t,\mathrm{e}}[t]$). For example, lag 1 sample ACF of the complete series can be extracted from the pairs $(x_1, x_2), (x_2, x_3), (x_3, x_4), (x_4, x_5), \ldots, (x_{n-2}, x_{n-1}), (x_{n-1}, x_n)$. The first lag ACF can again be extracted based on the *odd-even* cross correlation of the pairs $(x_1, x_2), (x_3, x_4), (x_5, x_6), \ldots, (x_{n-1}, x_n)$. Another approach is to estimate the same from *even-odd*(1 lag of odd) cross correlations, using the pairs $(x_2, x_3), (x_4, x_5), (x_6, x_7), \ldots, (x_{n-2}, x_{n-1})$. If the two series exhibit consistent sample ACFs, then one of these estimates gives sample ACFs of the complete series.

The consistent behaviour of the two estimates from odd and even series may not hold for short length series due to sampling error. Using simulated time series data of known linear time series models, the results found that this property holds well for very long series ($n > 10000$), but the estimates were found to be inconsistent for short series ($n < 100$). Figure 7.3 given an example for ACFs at even lags of the complete series from its odd and even series. Randomly generated data from MA(2) model was considered for complete series with various sample lengths and, sample ACFs are obtained from its odd-even series. Figures show that sample ACF values of both odd and even series are getting closer and equal to the actual sample ACF value of the complete series when sample size increases.

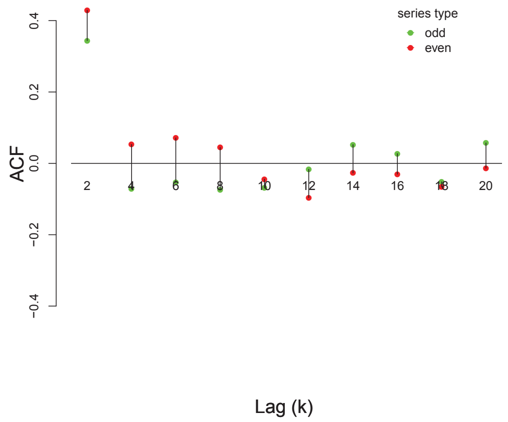
Figure 7.3: Sample ACFs at even lags of complete series from odd and even series: complete series is equivalent to MA(2):$\theta_1 = 0.3, \theta_2 = 0.6$, ACF at lag 2≈4.1



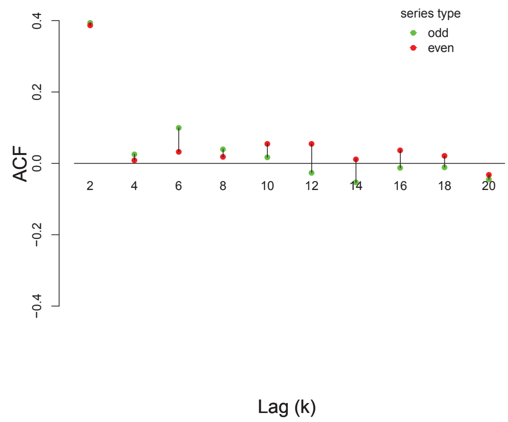
(a) $n = 100$



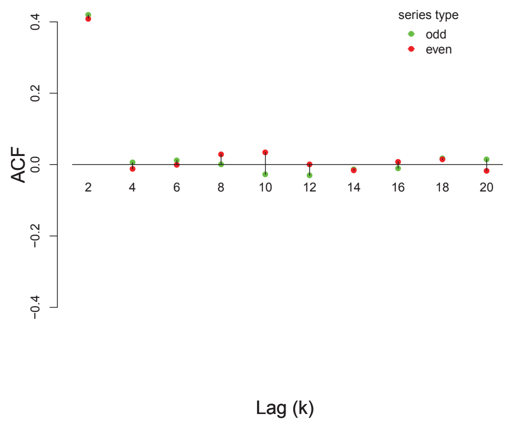
(b) $n = 500$



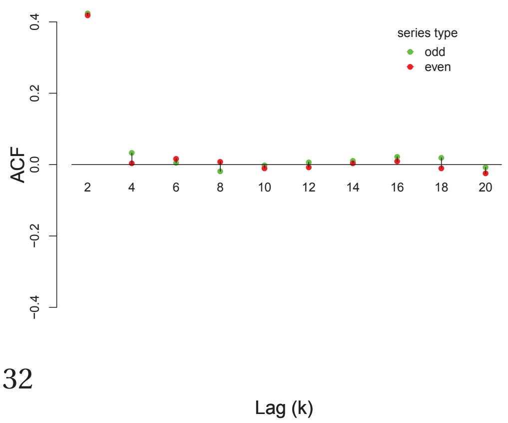
(c) $n = 1000$



(d) $n = 2000$



(e) $n = 5000$



(f) $n = 10000$

7.4 Summary

In this chapter, the autocorrelation and partial autocorrelation functions of the odd-even splits of popular linear time series models (AR(p), MA(q) and ARMA(p, q)) were derived. Results derived in this chapter have shown that the odd-even series always show less autocorrelation than the complete series, therefore these findings are useful for other time series applications. Sample autocorrelation functions can be re-calculated for the complete series from its odd and even series. The ACFs at the even lags for the complete series can be estimated using the ACFs in odd and or even series, and cross-correlations between the odd and even series are required for the ACFs of complete series at odd lags. However, consistency of these new estimates only holds for large sample sizes.

Analytically both odd and even series must have the same autocorrelation structure; therefore, examination of the autocorrelation structure for the split series and comparing the results with the pattern seen in the complete series is proposed as a useful exploratory tool. The use of the odd-even split approach for the analysis of stock returns will be demonstrated in Chapter 8 to evaluate the significance of autocorrelation in S&P 500 stock returns.

Chapter 8

Autocorrelations in stock returns

In this chapter, the significance of autocorrelation in stock returns is examined by using the odd-even split approach. The findings in this study rely on the expectation that these odd and even series should exhibit the same autocorrelation properties. It is argued that if a stock shows inconsistent results for the odd and even splits, then its complete series probably suffers from the impact of price shocks due to non-recurrent factors. The true autocorrelation in returns is assessed after adjusting for *induced* autocorrelation and outliers. The chapter is devoted to address *Proposition* 3: Stock returns will show zero autocorrelation in the common cause periods and *Issue* 3. Much of this chapter form part of Premarathna et al. (2016a).

8.1 Introduction

Debates on the reasons for autocorrelation in stock returns and its interpretation are continuing in the business and finance literature. One area of disagreement is the economic relevance versus statistical significance of observed autocorrelation in stock returns. For example, Fama (1970) found that a large proportion of daily stock returns exhibit positive (but not significant) autocorrelation; French and Roll (1986) discovered significant positive autocorrelation at the first lag and negative autocorrelation at higher lags; Fama (1991) claimed that the presence of autocorrelation is economically insignificant; and, Campbell et al. (1997) showed that the autocorrelation in weekly stock returns is weak and negative.

Many stock returns exhibit non-zero autocorrelation and partial autocorrelation function (ACF and PACF) values at lag 1. Fama (1991) showed an example of returns showing an ACF of -0.5, when the underlying price series is AR(1). Part of this autocorrelation is induced because returns are calculated as differences of consecutive terms in the logarithmic price series. This issue is discussed in Section 8.2 using known linear and non-linear time series models.

A list of tests which are used to test weak-form market efficiency can be found in Table 2 of Khan and Vieito (2012) including the autocorrelation test. Outliers are repeatedly present in series of stock returns and their presence can alter the true autocorrelation structure; see Chernick et al. (1982). Non-linearity in stock return series affects for the autocorrelation test of market efficiency (see Lim and Brooks (2010) and the references therein).

From Shewhart's philosophical view, there is a potential for the unusual observation to mask real autocorrelation patterns in stock returns. Reducing their impact is therefore investigated for autocorrelation in stock returns. A simple procedure of partitioning the complete series into two disjoint series based on the odd or even time index, referred to as the *odd-even split* (from Chapter 7) is used to identify the induced autocorrelation. Details are given in Section 8.3.

The remainder of the chapter is organized as follows. In Section 8.4, autocorrelation for stock returns in S&P 500 stocks are tested using a bootstrap method. Then, the odd-even split is used to asses the autocorrelation in returns and market efficiency. Section 8.5 summarises the chapter.

8.2 Illustration of induced ACFs/PACFs

Stock returns are usually computed as follows. Let S_t be the price of a stock recorded at time (say day) t. Stock returns r_t for day t is defined as,

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{8.1}$$

Given $r_{t+1} = \ln\left(\frac{S_{t+1}}{S_t}\right)$, the price S_t is included in both r_t and r_{t+1}. As a consequence, r_{t+1} becomes a function of r_t and hence the return series will exhibit lag 1 dependency. The autocorrelation function ρ_k at lag k is defined as,

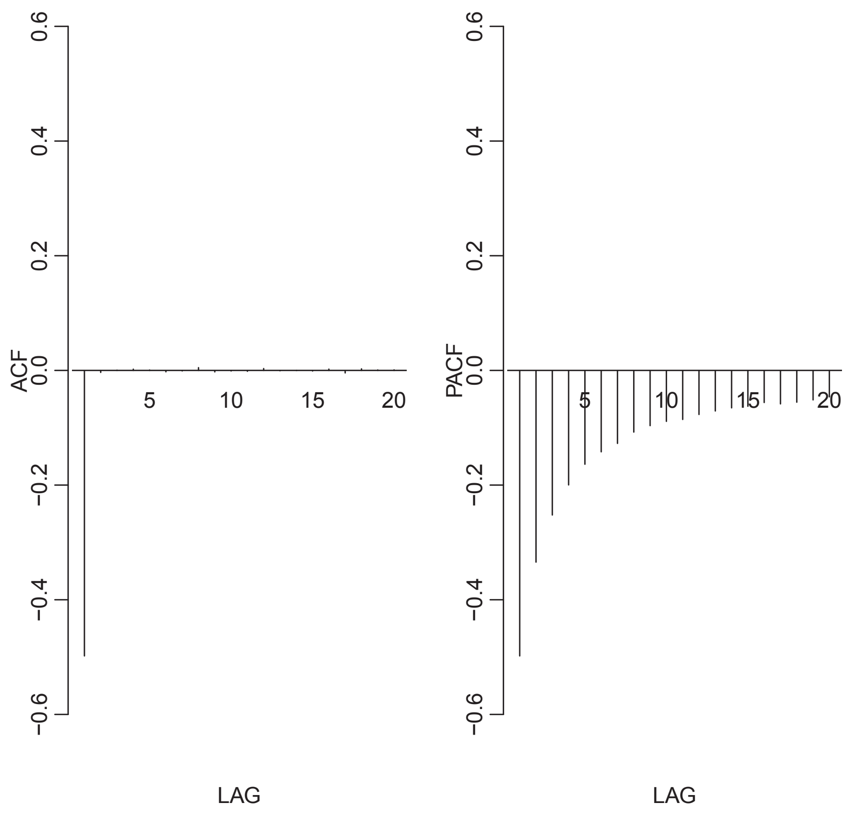
$$\rho_k = \frac{\mathrm{E}\left[\left(r_t - \mu\right)\left(r_{t+k} - \mu\right)\right]}{\sqrt{\mathrm{E}\left[\left(r_t - \mu\right)^2\right]\mathrm{E}\left[\left(r_{t+k} - \mu\right)^2\right]}} \tag{8.2}$$

where, μ is the mean of the series; r_t $(t = 1, 2, \ldots, n)$. The partial autocorrelation ϕ_{kk} between r_t and r_{t-k} is defined as the conditional correlation between r_t and r_{t-k}, conditional on $r_{t-k+1}, \ldots, r_{t-1}$, the data in the series between the time points t and $t-k$.

8.2.1 Linear time series models and induced ACFs

White noise (WN) series and several linear stationary time series models were considered for the underlying stock price series. Formal descriptions of these models were given in Section 7.2 (Table 7.1). The WN process is artificial for a price series. The significance of sample ACF/PACF for log returns corresponding to the above price series models can be studied using randomly generated data of those models. The null hypothesis of $H_0 : \rho_k = 0$ vs. $H_1 : \rho_k \neq 0$ is tested at various lags for the above price series models. The built-in functions *arima.sim* and *acf* (*pacf*) in the R statistical package were used to generate the data for the underlying price series, and to calculate sample ACF (PACF) values respectively.

Figure 8.1: ACFs/PACFs of log returns for a WN price series



If the underlying price series is a WN process, the complete return series show the lag 1 ACF of about -0.5 with PACF values decaying to zero. Clearly this lag 1 ACF is induced by the definition of the return (Equation 8.1). The ACF/PACF of log returns of a WN process is also similar to the MA(1) process which is shown in Figure 8.1.

If the underlying price series is MA(1), then the return series become a MA(2) process, and two scenarios for MA(2) process are found depending on the value of θ_1 (see Figure 8.2 and 8.3).

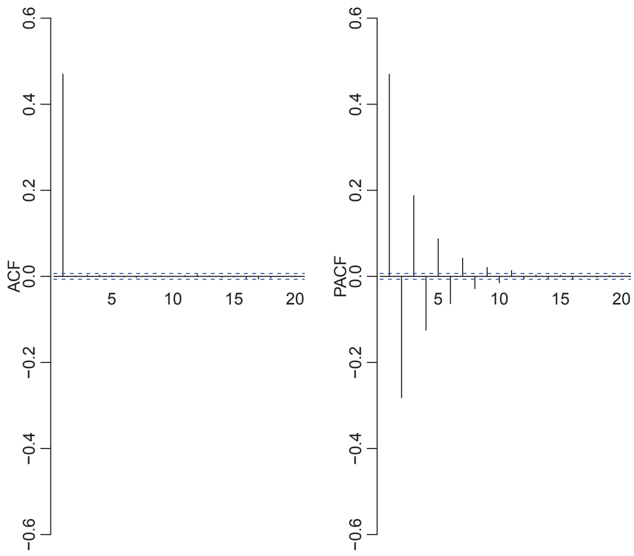
If the underlying price series of AR(1), then the return series is equivalent to the ARMA(1,1) process (see Figures 8.4 and 8.4 for $\phi_1 < 0$ and $\phi_1 > 0$ respectively). Clearly, the underlying structure of ACF/PACF in the price series and the magnitude of their

Figure 8.2: ACFs/PACFs of MA(1) price series and ACFs/PACFs of log returns, where $-1 < \theta_1 < 0$

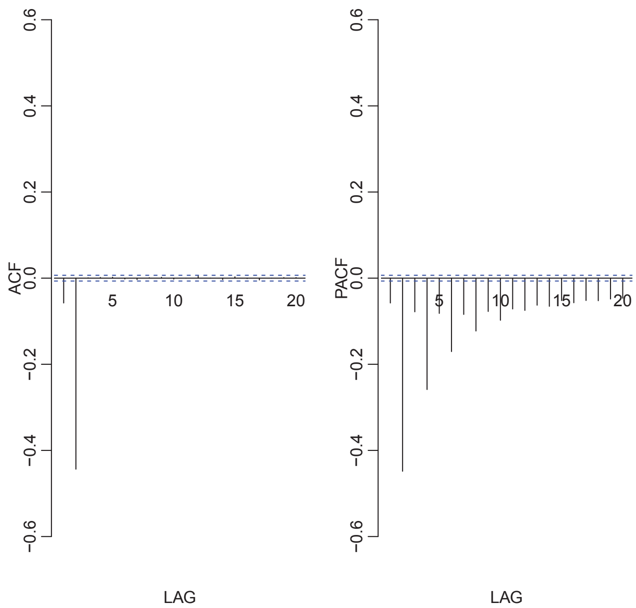
price series: MA(1)

log returns

Figure 8.3: ACFs/PACFs of MA(1) price series and ACFs/PACFs of log returns, where $0 < \theta_1 < 1$



price series: MA(1)



log returns

Table 8.1: Autocorrelation properties of the complete series and log returns. Liner time series models were assumed for the price series (complete series)

series	complete series (price series)	log-returns
White noise	WN	MA(1) $\rho_1 \approx -0.5$
MA(1)	$\rho_1 = \frac{\theta_1}{1+\theta_1^2}$ $\rho_k = 0, k \geq 2$	MA(2) $\rho_1, \phi_{11} \approx \frac{-(\theta_1-1)^2}{2(\theta_1^2-\theta_1+1)}$ $\rho_2 \approx \frac{-\theta_1}{2(\theta_1^2-\theta_1+1)}$
AR(1)	$\rho_1 = \phi_1$	ARMA(1,1) $\rho_1, \phi_{11} \approx \frac{\phi_1 - 1}{2}$ $\rho_2 \approx \frac{\phi_1(\phi_1-1)}{2}$

parameters affects the patterns of ACF/PACF in log returns. Table 8.1 summarises the ACFs in stock returns for the price series of WN, MA(1) and AR(1) models.

8.2.2 Non-linear time series models and induced ACFs

Non-linear time series models are often used to describe and forecast the mean and volatility in financial series (Franses and Van Dijk, 2000). Two non-linear time series models – the ARCH (autoregressive conditional heteroskedasticity) proposed in Engle (1982) and GARCH (generalised ARCH) proposed by Bollerslev (1986) are considered to investigate the impact of non-constant variability in price series on the return series. A mixture of the ARCH/GARCH models for conditional variance and the ARMA specification for the conditional mean includes attractive features that can be used to describe the momentum in financial series (Lange et al., 2011; Francq and Zakoïan, 2004). The following equations describe the ARMA-GARCH model (see Francq and Zakoïan, 2004, p.611).

ARMA(p,q) – **GARCH**(P,Q) process

$$Y_t = \mu_0 + \sum_{i=1}^{p} \phi_i Y_{t-i} + \epsilon_t - \sum_{j=1}^{q} \theta_j \epsilon_{t-j}$$

$$\epsilon_t = \sqrt{h_t \eta_t}$$

$$h_t = \omega_0 + \sum_{i=1}^{Q} \alpha_j \epsilon_{t-j}^2 + \sum_{j=1}^{P} \beta_j h_{t-j}$$

The R statistical package "rugarch" Ghalanos (2014) was used to study the autocorrelation behaviour for a selection of ARMA(p,q)-GARCH(P,Q) processes. When the underlying price series is ARMA-GARCH, the induced ACF/PACFs of stock returns are similar to the behaviour of ACF/PACF for returns from linear ARMA models (those with the same AR and MA specifications). Based on the price models considered, the issue of non-constant variability does not seem to induce or affect ACF/PACF any differently to the simpler models. In the next section, the odd-even split of the complete return series is used to discuss ACF/PACF properties further because this approach helps to exclude the induced effects.

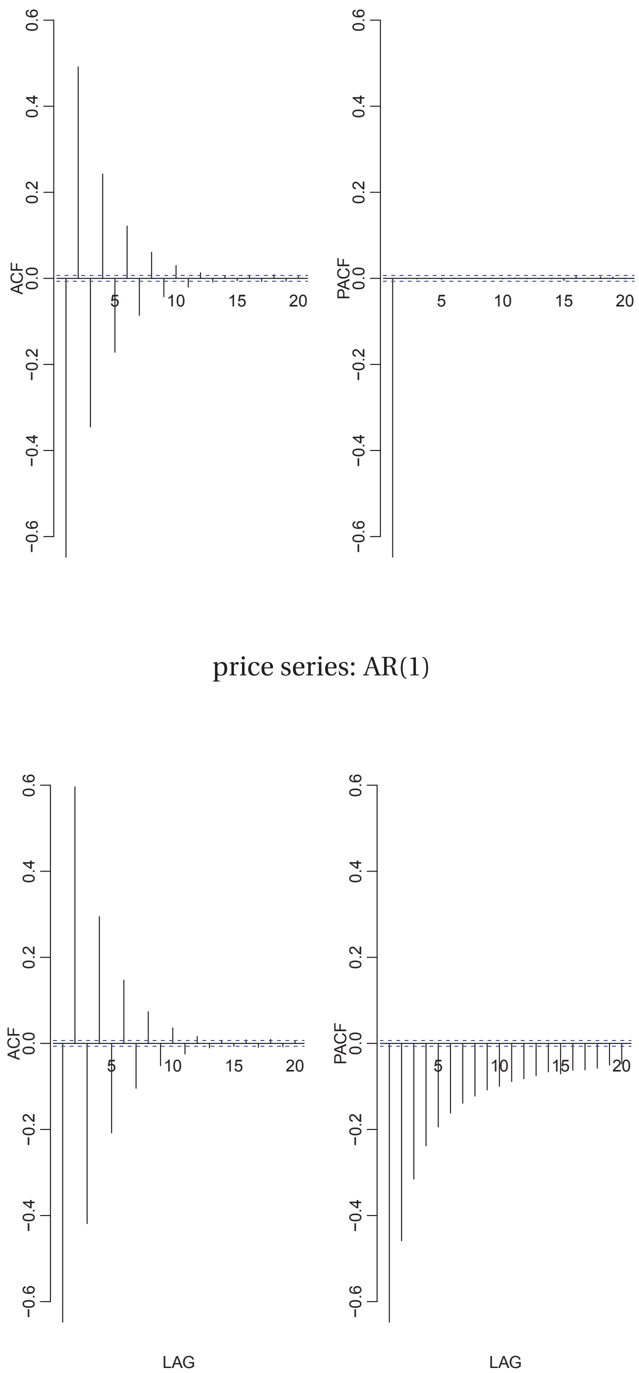
8.3 The odd-even split and identification of induced autocorrelation

Induced autocorrelation in stock returns was demonstrated in the previous section using known time series models for price series, and Chapter 5 derived the odd-even split for ARMA(p,q) models. It is well known that sample autocorrelation estimates are not robust and are affected by outliers. Induced autocorrelation may also be caused by outliers. If the underlying price series is known, the induced ACF in stock returns can be estimated but identification of the true model is difficult when working with real (sampled) data which are invariably contaminated by special causes.

A series of stock returns of a linear time series models should yield equal ACFs in its odd and even series (Section 7.2). Therefore, the odd-even split can be used to identify the effect of contamination due to special causes in returns. If the odd-even split of returns leads to any inconsistency in the ACF/PACF values, then it suggests that special causes exist in the complete series.

Section 8.2 showed that return autocorrelations can be induced by the autocorrelation structure of the price series. As stated in Chapter 1, the efficient market hypothesis implies that the price series follows a random walk model, therefore returns approximately follow a white noise process. Then autocorrelations in complete return series (white noise series) and its odd-even series are not significant. In this chapter, this property is used to test market efficiency in terms of autocorrelation in actual return series after reducing the effect of possible outliers.

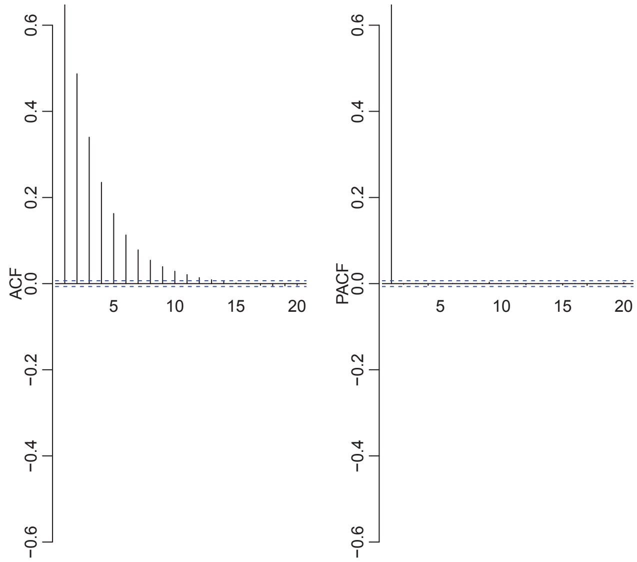
Figure 8.4: ACFs/PACFs of AR(1) price series and ACFs/PACFs of log returns, where AR(1): $-1 < \phi_1 < 0$



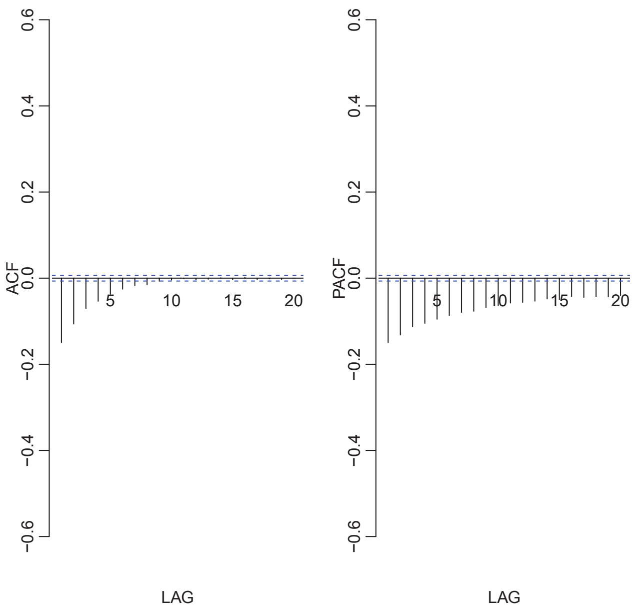
price series: AR(1)

log returns

Figure 8.5: ACFs/PACFs of AR(1) price series and ACFs/PACFs of log returns, where AR(1): $0 < \phi_1 < 1$



price series: AR(1)



log returns

8.4 Empirical analysis

Daily adjusted closing prices of stocks in S&P 500 for six years (January, 2001 to December, 2015: approximately 3800 observations) were used. Log returns (complete series) are calculated and then partitioned them based on the odd and even indexes.

The use of normal distribution approximation for the distribution of the test statistics for the null–hypothesis of zero ACF/PACF is avoided and opted for non–parametric bootstrap methods. Bootstrapping in time series was addressed by the moving block bootstrap (MBB) method in Kunsch (1989) and Liu and Singh (1992) to preserve the dependency structure. The MBB considers re-sampling of overlapping blocks. The parameter required is the block length and it should be sufficient to preserve the original dependence. According to Ju (2015), if the dependence is weak, then the approximation is better when the blocks are as long as possible. In the MBB approach, the blocks are sampled and combined, in contrast to taking a single observation at a time. Lahiri (1999) compared the common block bootstrap methods and has shown that the overlapping blocks and non-random block lengths are suitable. A summary of the method is given below, quoted from Lahiri (2013).

The MBB procedure can be described as follows. If the return series $R_t : \{r_1, r_2, r_3, ..., r_{N_i}\}$ is k^{th} lag dependent, then the moving blocks of size k is selected. The corresponding moving block of size k, denote by B_i, where $B_i = \{r_i, r_{i+1}, r_{i+2}, ..., r_{i+k-1}\}$ for $i = 1, ..., N - k + 1$. In the MBB procedure, n moving blocks from the set of $\{B_1, B_2.., B_{N-k+1}\}$, are sampled with replacement. The bootstrap sample under MBB is such that, $\left\{r_1^*, r_2^*, r_3^*, ..., r_{N_i}^*\right\} \equiv \{b_{11}, ...b_{1k}, ..., b_{n1}, ...b_{nk}\}$ (where b is selected in a such way that $k = Nb$) blocks are drawn and denoted by $b_1, ...b_n$ and each of b_i consist of k elements, which can be written as $b_i \equiv (r_{i1}, ...r_{ik})$. ACF values for each bootstrap sample are calculated, and the sampling distribution of ACF values was considered to obtain the critical limits based on a given level of α.

The two functions: *movingBlocks* and *getPositions* in the R-package “quantspec" (Kley, 2016) were used to generate an index matrix of 2000 moving blocks bootstrap samples. The actual data were extracted corresponding to the index matrix. Lower and upper quantiles of the sampling distribution for ACF/PACF at lag k of the complete series were used to determine the significance of ACF/PACF for the complete series. The quantiles established for even lags of the complete series ($w = 2, 4, 6, \ldots$) were then used as the quantiles for the odd and even series at all lags ($w/2$).

Table 8.2 gives the proportions of stocks showing significant ACF and PACF values in the complete, odd and even return series. The empty cells in “odd" and “even" columns can be filled by using cross-correlations between odd and even series. It can be noted that the first lag ACF is statistically significant for 161 (39.4%) out of 409 stocks; 145(35%) of stocks in the complete series show a negatively significant ACF at lag 1; 25% of stocks show a significant ACF at lag 2; 11% of stocks at lag 3; 35 (8.5%) of stocks show a significant ACF at both lag 1 and lag 2 for the complete series. Surprisingly the number of stocks with significant ACF values does not decrease even at higher lags. This ACF/PACF behaviour may be interpreted as a symptom of market inefficiency. The ACF/PACF values and their significance for the top 10 highly market capitalized S&P 500 stocks are shown as Table 8.3 and Table 8.4. For these ten stocks, PACF values and their significance are largely equivalent to that of ACFs. Bold font is used to indicate the significance at 5%.

The Ljung–Box test (LB test) (Box and Pierce, 1970; Ljung and Box, 1978) is an omnibus test to examine the overall or cumulative effect of autocorrelations up to a given (k^{th}) lag. The significance of the LB test statistic is also examined for $k = 2, \ldots, 15$ at a 5% significance level. The empirical distribution of the LB test statistic and the 0.975 and 0.025 critical quantile limits were obtained using the bootstrap method. The proportions of stocks that give significant ACF values up to the k^{th} lag based on the LB test are given along with proportions of ACF in Table 8.2. For the complete series, 31% to 40% of stocks

Table 8.2: Proportions of stocks with significant ACF/PACF in complete, odd and even series in S&P 500 list. The columns under LB test give the proportions of stocks significant ACF/PACF from the Ljung-Box test

ACF								PACF				
Lag	**Total**	**LB Test**	**Lag**	**Odd**	**LB Test**	**Even**	**LB Test**	**Lag**		**Lag**	**Odd**	**Even**
1	39.36%							**1**	39.36%			
2	24.94%	40.10%	**1**	39.61%		32.03%		**2**	28.36%	**1**	39.12%	31.78%
3	10.76%	36.19%						**3**	9.29%			
4	9.29%	34.72%	**2**	17.11%	17.60%	31.30%	9.78%	**4**	9.54%	**2**	17.85%	32.03%
5	18.34%	36.43%						**5**	19.56%			
6	6.85%	34.96%	**3**	15.89%	18.83%	20.78%	12.96%	**6**	7.33%	**3**	16.63%	21.76%
7	8.31%	34.96%						**7**	8.56%			
8	10.51%	34.96%	**4**	33.99%	15.89%	18.34%	17.85%	**8**	9.05%	**4**	33.99%	19.80%
9	4.40%	33.50%						**9**	5.38%			
10	14.67%	33.74%	**5**	22.98%	18.83%	30.81%	20.78%	**10**	14.18%	**5**	21.76%	31.54%
11	10.02%	33.50%						**11**	9.05%			
12	6.60%	33.99%	**6**	20.54%	22.25%	12.47%	24.21%	**12**	6.11%	**6**	19.32%	13.20%
13	7.82%	32.27%						**13**	7.82%			
14	10.27%	31.54%	**7**	13.45%	25.43%	24.69%	32.03%	**14**	7.58%	**7**	12.96%	22.74%
15	10.51%	32.52%						**15**	10.27%			
16	21.27%	34.96%	**8**	58.19%	23.96%	20.29%	35.94%	**16**	17.60%	**8**	57.95%	20.54%

showed significant overall autocorrelation effects based on the LB test. For the odd series, the LB test statistic showed significance for 15% to 23% of the stocks, and this statistic was significant for 10% to 35% cases of even series of stocks. The odd and even series particularly showed inconsistent LB test results even for lags 2 to 8.

In the absence of outliers, the odd-even split is expected to lead to an approximately equal proportion of significant autocorrelations at lag 1; this should also be approximately equal to that for the complete series at lag 2. For this empirical data, the proportions in odd, even and complete series are 40%, 32% and 25% respectively (from Table 8.2) respectively. So there is a substantial inconsistency in the autocorrelation structures of the complete series when compared to the patterns found for the two split series. A similar pattern is also observed for lag 2 in the odd-even series and lag 4 in the complete series. In Table 8.5, the diagonal elements correspond to significant and insignificant proportions of stocks in both odd and even series respectively; off-diagonal entries give the proportions of stocks for which significance is not observed in both odd and even series; results are given for lags 1 to 4, for example 11.25% stocks show lag 1 significant ACF in both odd and even series. This table shows that about 48% of stocks do not show consistent significance results at lag 1 when complete and split series are compared for concordant autocorrelation behaviour.

The plots of odd vs even ACF values for lags 1-4 are given in Figure 8.6. For 409 of S&P 500 stocks examined, points are expected to plot evenly in the four quadrants along the 45° line when the odd and even series are concordant in terms of their autocorrelations. The odd and even ACF values are expected to lie on the line of $x = y$. The stocks that lie close to this line are the most concordant ones; the further a point lies from the $x = y$ line, the more discordant that stock's results are. Stocks with the greatest discordance would lie in the north-west and south-east corners of Figure 8.6: these stocks have significant lag k positive ACF values in the odd series and negative ACF values in the even series or vice

Table 8.3: ACF values of complete, odd and even series for publicly traded companies having the greatest market capitalization, bold font is used to indicate the significance at 5%

		ACF					
Ticker	**Series**	**Lag 1**	**Lag 2**	**Lag 3**	**Lag 4**	**Lag 5**	**Lag 6**
AAPL	Complete	-0.0106	**-0.0506**	0.0012	**0.0453**	-0.0054	0.0064
	Odd		**-0.0450**		0.0382		0.0293
	Even		**-0.0570**		**0.0528**		-0.0193
BAC	Complete	-0.0121	0.0384	**-0.0471**	0.0027	**-0.1078**	-0.0246
	Odd		-0.0308		-0.0254		0.0145
	Even		**0.1106**		0.0315		**-0.0666**
BMY	Complete	-0.0197	-0.0149	-0.0006	-0.0211	-0.0209	0.0300
	Odd		-0.0133		0.0020		0.0283
	Even		-0.0173		**-0.0456**		0.0311
BRK-B	Complete	-0.0164	0.0157	**-0.0530**	**-0.0529**	**-0.0885**	-0.0124
	Odd		0.0247		-0.0241		0.0054
	Even		0.0079		**-0.0770**		-0.0274
C	Complete	**0.0631**	0.0401	-0.0329	**-0.0780**	0.0172	-0.0155
	Odd		-0.0207		**-0.1251**		0.0298
	Even		**0.1165**		-0.0194		**-0.0745**
GE	Complete	-0.0077	0.0056	-0.0166	0.0073	**-0.0568**	0.0332
	Odd		0.0089		0.0385		**0.1239**
	Even		0.0016		-0.0217		**-0.0489**
JNJ	Complete	**-0.0434**	**-0.0763**	-0.0061	-0.0086	-0.0202	0.0255
	Odd		**-0.1059**		0.0367		0.0383
	Even		**-0.0438**		**-0.0586**		0.0113
JPM	Complete	**-0.0825**	-0.0219	-0.0293	-0.0249	-0.0371	-0.0180
	Odd		-0.0322		-0.0229		0.0233
	Even		-0.0113		-0.0280		**-0.0646**
PFE	Complete	-0.0246	**-0.0811**	0.0169	-0.0095	-0.0130	-0.0033
	Odd		**-0.1281**		0.0289		0.0127
	Even		-0.0356		**-0.0470**		-0.0189
XOM	Complete	**-0.1095**	**-0.0838**	**0.0457**	-0.0147	-0.0403	-0.0008
	Odd		**-0.1617**		**0.0854**		-0.0056
	Even		-0.0042		**-0.1177**		0.0039

AAPL: Apple Inc., BAC: Bank of America Corp, BMY: Bristol-Myers Squibb,
BRK-B: Berkshire Hathaway, C: Citigroup Inc., GE: General Electric,
JNJ: Johnson & Johnson, JPM: JPMorgan Chase & Co., PFE: Pfizer Inc.,
XOM: Exxon Mobil Corp.

Table 8.4: PACF values of complete, odd and even series for publicly traded companies having the greatest market capitalization, bold font is used to indicate the significance at 5%

		PACF					
Ticker	**Series**	**Lag 1**	**Lag 2**	**Lag 3**	**Lag 4**	**Lag 5**	**Lag 6**
AAPL	Complete	-0.0106	**-0.0507**	0.0001	0.0429	-0.0044	0.0107
	Odd		**-0.0450**		0.0363		0.0327
	Even		**-0.0570**		**0.0497**		-0.0137
BAC	Complete	-0.0121	0.0382	**-0.0463**	0.0002	**-0.1046**	-0.0294
	Odd		-0.0308		-0.0263		0.0129
	Even		**0.1106**		0.0195		**-0.0730**
BMY	Complete	-0.0197	-0.0153	-0.0012	-0.0214	-0.0218	0.0285
	Odd		-0.0133		0.0018		0.0283
	Even		-0.0173		**-0.0460**		0.0295
BRK-B	Complete	-0.0164	0.0155	**-0.0525**	**-0.0550**	**-0.0892**	-0.0175
	Odd		0.0247		-0.0247		0.0067
	Even		0.0079		**-0.0771**		-0.0263
C	Complete	**0.0631**	0.0363	-0.0379	**-0.0756**	0.0296	-0.0133
	Odd		-0.0207		**-0.1256**		0.0247
	Even		**0.1165**		-0.0334		**-0.0693**
GE	Complete	-0.0077	0.0056	-0.0165	0.0070	**-0.0565**	0.0321
	Odd		0.0089		0.0384		**0.1234**
	Even		0.0016		-0.0217		**-0.0489**
JNJ	Complete	**-0.0434**	**-0.0783**	-0.0132	-0.0157	-0.0230	0.0216
	Odd		**-0.1059**		0.0258		**0.0454**
	Even		**-0.0438**		**-0.0606**		0.0059
JPM	Complete	**-0.0825**	-0.0289	-0.0339	-0.0312	-0.0441	-0.0280
	Odd		-0.0322		-0.0239		0.0218
	Even		-0.0113		-0.0281		**-0.0653**
PFE	Complete	-0.0246	**-0.0817**	0.0129	-0.0155	-0.0113	-0.0062
	Odd		**-0.1281**		0.0127		0.0183
	Even		-0.0356		**-0.0483**		-0.0225
XOM	Complete	**-0.1095**	**-0.0970**	0.0256	-0.0147	-0.0381	-0.0140
	Odd		**-0.1617**		**0.0608**		0.0177
	Even		-0.0042		**-0.1177**		0.0029

AAPL: Apple Inc., BAC: Bank of America Corp, BMY: Bristol-Myers Squibb,
BRK-B: Berkshire Hathaway, C: Citigroup Inc., GE: General Electric,
JNJ: Johnson & Johnson, JPM: JPMorgan Chase & Co., PFE: Pfizer Inc.,
XOM: Exxon Mobil Corp.

Table 8.5: Proportions of significant/insignificant ACF/PACF; odd vs. even for lags 1-4

		ACF		**ACF**	
		Odd series		**Odd series**	
		Lag 1		**Lag 2**	
		Sig	**Non-sig**	**Sig**	**Non-sig**
Even series	**Sig**	11.25%	20.78%	4.65%	26.65%
	Non-sig	28.36%	39.61%	12.47%	56.23%
		Lag 3		**Lag 4**	
		Sig	**Non-sig**	**Sig**	**Non-sig**
Even series	**Sig**	4.40%	16.38%	8.31%	10.02%
	Non-sig	11.49%	67.73%	25.67%	55.99%

versa. Discordance can be measured using $|x - y|$, where x and y are the ACF values of the odd and even series at the chosen lag; this is the perpendicular distance from the point for each stock to the line $x = y$.

8.4.1 Effect of outliers

The presence of extreme outliers in the complete series is manifested as inconsistent sample ACF/APCF estimates in its odd and even series. In order to remove the outlier effect on the sample ACF/PACF, the following three methods are tried:

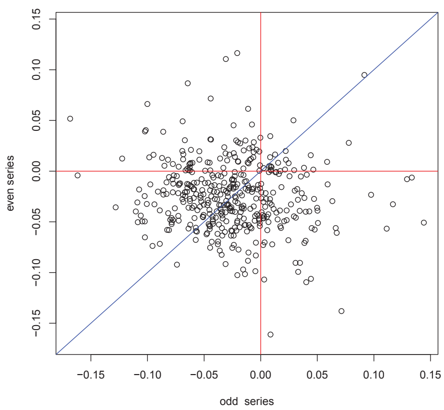
Method 1: Returns in the complete series which are above the $Q3$+1.5IQR or below the $Q1$-1.5 IQR were replaced by their fence values, where $Q1$: and $Q3$: are the lower and upper quartiles and the IQR is the difference between them.

Method 2: Returns in the complete series which are above the $Q3$+1.5IQR or below the $Q1$-1.5 IQR were replaced by $Q2$+0.5IQR and $Q2$-0.5IQR respectively.

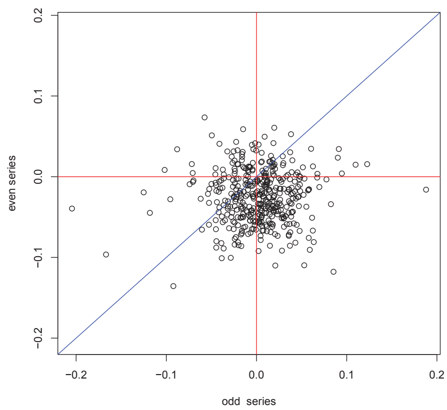
Method 3: Trimming 10% of the complete return series.

Let us take the complete return series as r_t and $r_{t_{m_1}}, r_{t_{m_2}}, r_{t_{m_3}}$ be the return series after applying Method 1, Method 2 and Method 3 respectively. Figure 8.7 gives the lag 1 ACF values for r_t series vs. lag 1 ACF values for the three series $r_{t_{m_1}}, r_{t_{m_2}}$ and $r_{t_{m_3}}$. Method 3

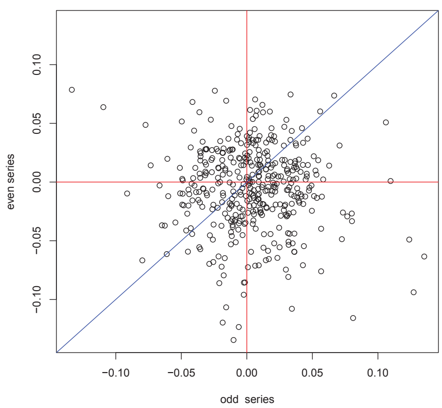
Figure 8.6: Comparison of lags 1-4 ACFs of odd vs even series of the original return series



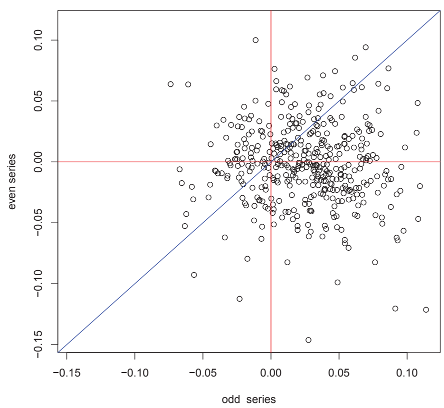
(a) lag 1



(b) lag 2



(c) lag 3



(d) lag 4

gives random behaviour of the ACF values at lag 1 compared to Method 1 and Method 2. In Method 1, the magnitude of the outliers was reduced and in Method 2, the outliers are replaced by a systematic approach. The elimination of the effect of market shocks (using these trial methods) supports our view that inconsistent ACF values in odd and even series signal the impact of outliers on the complete series. In addition, the significance of the ACF/PACF values in $r_{t_{m_3}}$ is investigated .

After trimming (Method 3), 14% of stocks still showed significant ACFs at lag 1 in the complete series. Only 1 to 4% of stocks show a significant ACF at lag 2 and onwards (see Table 8.6) 11% stocks show a lag 1 significant ACF in the complete series but not in both their odd and even series. However, an odd-even mismatch is still observed with complete series, for example, about 3% stocks show a significant ACF at lag 2, but the even and odd series give about 10% and 9% respectively. If the complete series has only induced autocorrelation, then its odd and even series should be uncorrelated. Very similar results are also noted for the PACFs of stocks.

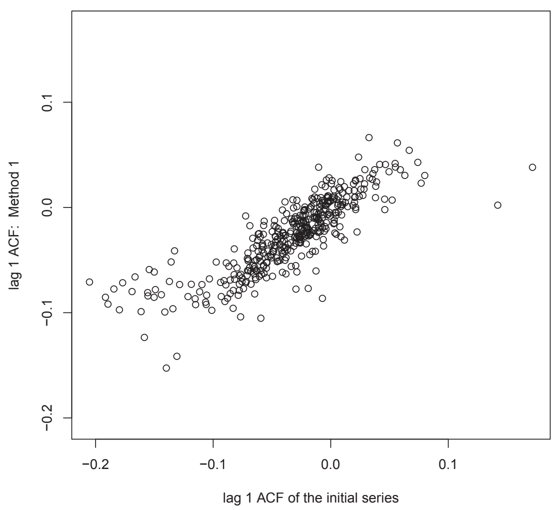
In Figures 8.7a, 8.7b and 8.7c, r_t denotes the complete return series and $r_{t_{m_1}}, r_{t_{m_2}}, r_{t_{m_3}}$ denote the three series after applying Method 1, Method 2 and Method 3 respectively. Method 1 reduces the magnitude of the outliers; As we can see in Figure 8.7a, the discordance has changed little (the points lie near $x = y$). In Figure 8.7b, Method 2 shows less discordance as we do not observe any point in the upper half of this figure. Using Method 3, where 10% of data were trimmed, does change the discordance and has a similar impact to Method 2, see Figure 8.7c. Methods 2 and 3 show reduced levels of discordance between the odd and even series.

In this chapter, the control chart techniques for partitioning common and special causes were not used to analyse the autocorrelation. Instead of identifying special cause periods, their effect is reduced using Methods 1 to 3. In most cases, the results largely sup-

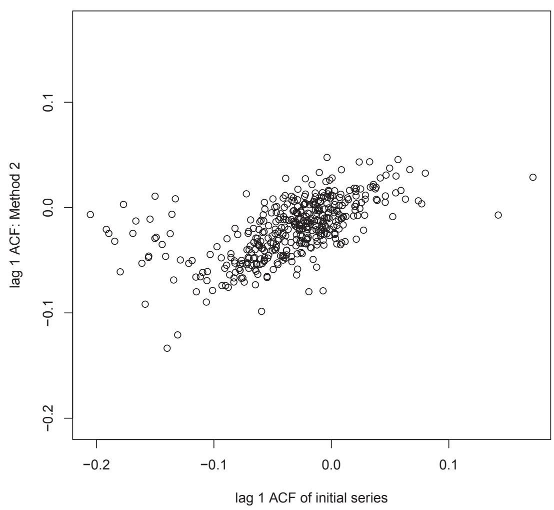
Table 8.6: Proportions of stocks with significant ACF/PACF in $r_{t_{m_3}}$ series and its odd and even series

ACF					PACF				
Lag	**Total**	**Lag**	**Odd**	**Even**	**Lag**		**Lag**	**Odd**	**Even**
1	13.94%				**1**	13.94%			
2	4.16%	**1**	11.49%	13.45%	**2**	4.16%	**1**	11.00%	13.69%
3	1.96%				**3**	1.96%			
4	3.18%	**2**	10.51%	9.05%	**4**	3.18%	**2**	10.51%	9.29%
5	1.47%				**5**	1.22%			
6	1.96%	**3**	10.76%	10.51%	**6**	1.71%	**3**	10.76%	9.29%
7	0.98%				**7**	1.47%			
8	0.98%	**4**	9.78%	8.56%	**8**	0.98%	**4**	9.54%	8.07%
9	2.20%				**9**	2.44%			
10	1.22%	**5**	10.76%	11.00%	**10**	0.98%	**5**	10.51%	11.00%
11	2.20%				**11**	2.44%			
12	0.73%	**6**	8.80%	11.00%	**12**	1.22%	**6**	10.02%	10.27%
13	0.98%				**13**	0.98%			
14	2.69%	**7**	9.54%	10.27%	**14**	2.93%	**7**	9.54%	10.27%
15	1.71%				**15**	2.20%			
16	1.47%	**8**	9.29%	10.02%	**16**	1.96%	**8**	9.05%	10.51%
17	2.44%				**17**	2.44%			
18	1.96%	**9**	8.80%	7.82%	**18**	1.71%	**9**	9.05%	9.05%
19	1.96%				**19**	2.44%			
20	1.96%	**10**	9.05%	10.27%	**20**	1.96%	**10**	8.07%	9.05%
21	1.71%				**21**	1.22%			
22	1.71%	**11**	9.54%	7.09%	**22**	1.71%	**11**	9.29%	8.07%
23	1.22%				**23**	1.22%			
24	1.71%	**12**	11.25%	7.82%	**24**	1.71%	**12**	11.00%	7.33%
25	1.22%				**25**	1.71%			
26	1.71%	**13**	9.05%	8.56%	**26**	1.71%	**13**	9.54%	9.05%

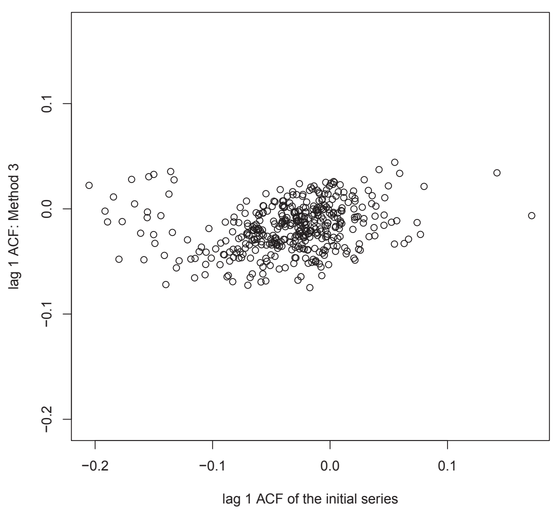
Figure 8.7: Comparison of ACF values at lag 1 for S&P 500 stocks after removing the effect of outliers



(a) Lag 1 ACF of r_t vs $r_{t_{m_1}}$



(b) Lag 1 ACF of r_t vs $r_{t_{m_3}}$



(c) Lag 1 ACF of r_t vs $r_{t_{m_3}}$

port *Proposition* 3 that the “stock returns (complete series) will show zero autocorrelation in the common cause periods".

The impact of different trading activities or the behaviour of short term investors might play a role in the true autocorrelation in stock returns, for example, Daniel et al. (1998) have explained how investors trading activities change the autocorrelation in stock returns. Several cases are therefore considered in this study.

8.4.2 Other effects

Some other reasons for the discordance of ACF values for the complete series are also investigated when compared to the corresponding ACF values for the odd and even series.

Day effect

An arbitrary starting date is selected for complete daily series of Friday, January 02, 2009 for the price series, so the complete series of returns series begins on Monday, January 05, 2009. There is empirical evidence for day effects in stock returns. French (1980) showed that the expected return for Monday is three times the expected return for other days of the week. However this analysis showed that the day effect on the ACFs of the odd and even series is not significant.

Effect of initial value

ACF values may be affected by the initial value of a series. In the initial analysis, all returns series start at Monday 5 January 2009. The data for the first month is deleted making Tuesday 3 February 2009 the start date and repeated the analysis. The results remained the same in both analyses.

Trading volume/Market capitalization

The effect of trading volumes and market capitalizations of the stocks are also considered and highlighted the discordant stocks. Neither variable was able to explain why the ACF values of the complete series are discordant to the ACF values of the odd-even series.

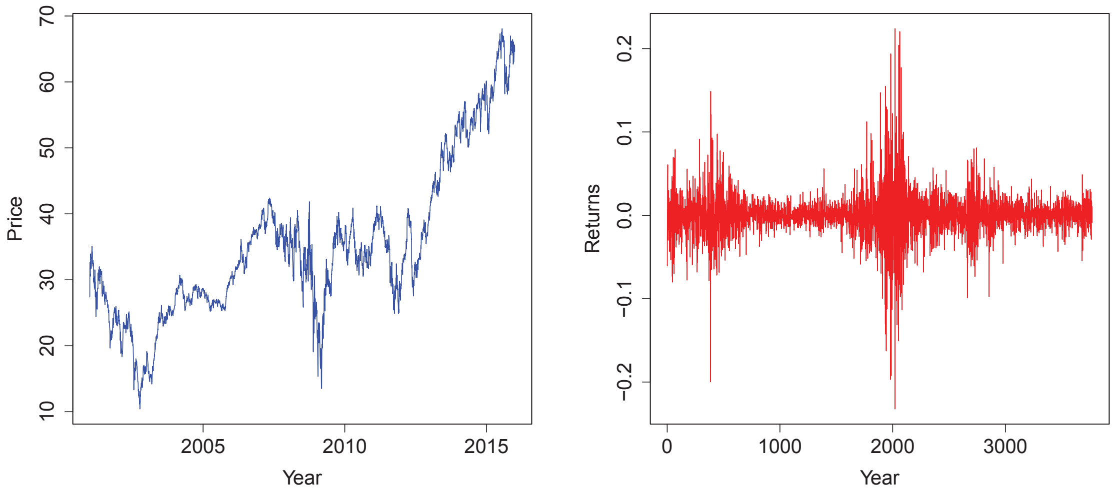
8.4.3 Autocorrelation test for market efficiency

Daily stock returns of JPMorgan Chase & Co. (ticker symbol: JPM) are considered to demonstrate the odd-even split for assessing market efficiency. The daily log returns from January 2001 to December 2015 were used. Figure 8.8 shows the price and return series over this period; returns exhibit unusual fluctuations.

Figure 8.9 shows the sample ACFs of three series. The significance limits are lower and upper quantiles of the relevant sampling distribution of ACFs. Figures 8.9b and 8.9c show inconsistent significance in odd and even series; in particular the even series shows a significant autocorrelation for the third lag and the odd series does not show any significant autocorrelation until eight lag; the complete series show a significant autocorrelation at first lag (see Figure 8.9a). For all three series, the sample PACFs are similar to their ACFs. It is found that the LB test statistic is significant for the complete series for the range of lags, $k = 2, ..., 15$ and the even series for the lags $k = 2, .., 7$. However the LB test was significant at lag 10 for the odd series.

In this example, the effect of unusual observations is removed using the Shewhart control chart procedure (details are given in Section 3.4), and then investigated the ACF/-PACFs. The complete series showed significant first lag ACF (Figure 8.10a) after eliminating the suspected special cause periods but not in its odd or even series. The only significant autocorrelation was at the first lag which casts suspicion of on the validity of the efficient market hypothesis for JPMorgan Chase & Co. stock returns. This example demonstrates that induced and true forms of autocorrelation in stock returns can be recognised using

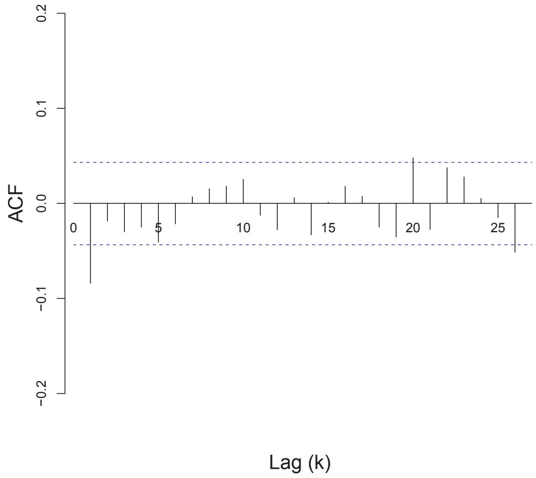
Figure 8.8: JPMorgan Chase & Co. (a) daily price series and (b) stock returns



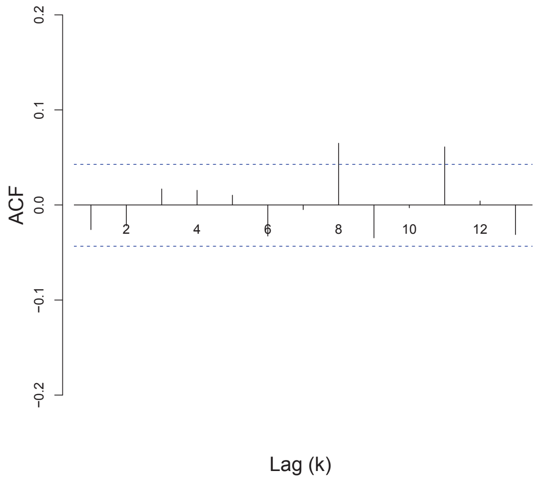
(a) stock prices **(b)** log returns

both Shewhart methodology and the odd-even split. Investigation of autocorrelation in all three series is important to discuss market efficiency.

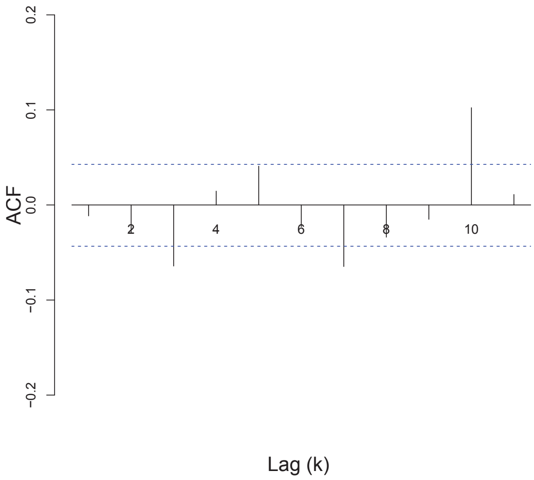
Figure 8.9: JPMorgan Chase & Co.: sample ACFs of stock returns of (a) complete series (b) odd series and (c) even series



(a) complete series

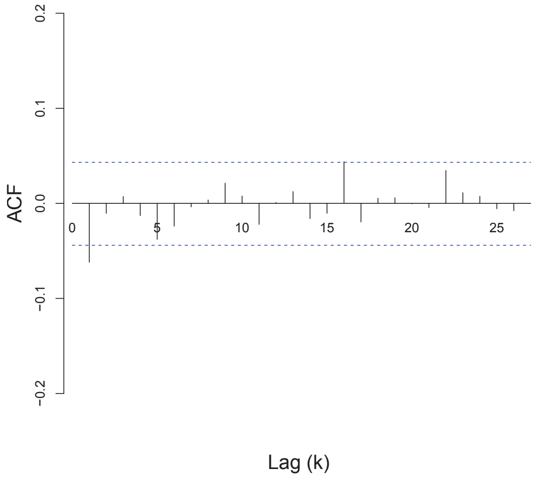


(b) odd series

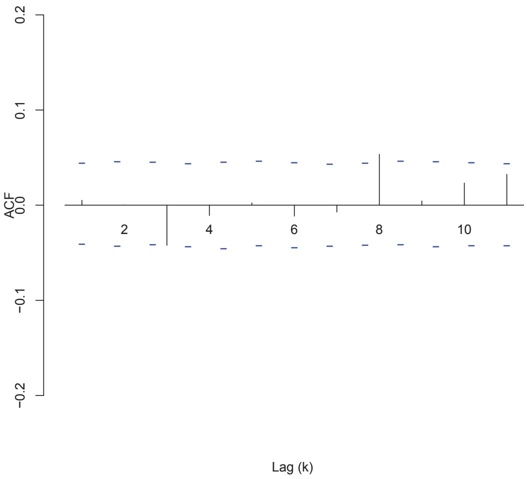


(c) even series

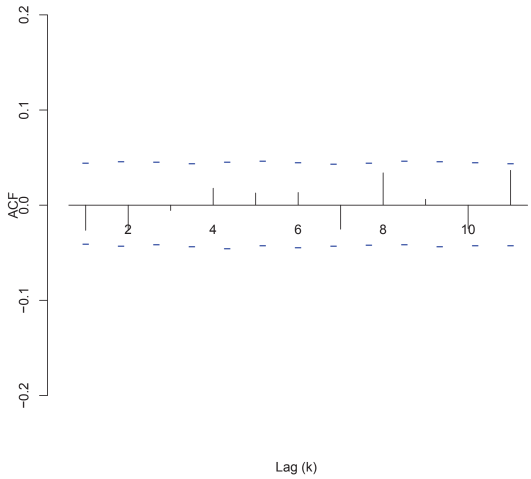
Figure 8.10: (a) sample ACFs of complete series of returns after eliminating outliers (b) odd series (c) even series



(a)



(b)



(c)

8.5 Summary

Induced autocorrelation is observed in stock returns due to the very definition of stock returns. However, all observed autocorrelations are not induced. This chapter used the odd-even split procedure (from Chapter 7) for two purposes; to identify the effect of outliers and to determined the induced and true components of autocorrelation in the return series. A large proportion of stocks in the S&P 500 list were found to be discordant in terms of the autocorrelation values in their odd and even series. This result is not only surprising but also interesting on theoretical grounds. The trimming approach was used to reduce the effect of special cause events, and it is found that ACFs in the trimmed data largely supports *Proposition* 3.

On the other hand, *Issue* 3 has been examined. This chapter showed that autocorrelation in stock returns depend on the autocorrelation structure in the underlying price series. If the theory of random walk is valid, then returns should exhibit zero autocorrelation. However, this chapter showed that extreme outliers masked the observed patterns of autocorrelations, and as a result, they contributed to a pattern of inconsistency in the odd and even split series. Autocorrelation in the common cause periods and its odd-even split can be used as a tool for further analysis of autocorrelation as a test of market efficiency.

Chapter 9

Applications

The Shewhart principles that have been used in this study have the potential for applications beyond undertaking the issues identified in Chapter 1. They can also be used as an investment analysis tool along with other strategies such as monitoring market behaviour. This chapter builds on the analysis of previous chapters to suggest some new aspects for stock returns analysis.

9.1 Applications for investors

Investors need to analyse the behaviour of financial assets that match their investing objectives. For example, are frequent and infrequent investors likely to take an equal risk on Google or Apple stocks? The answer to this question partly depends on the persistent and transient measures of observed volatility. Market participants always need to examine the way that expected return of an investment changes over time, and need to optimise the performance of the stocks in their portfolios depending on the risk that they are willing to take. To select appropriate stocks, investors use the risk/return trade–off based on the assertion that higher risk result in a greater return.

Andersen and Bollerslev (1998); Poon and Granger (2003) found that the squared returns based estimate is a very noisy proxy for volatility as financial returns display temporal dependencies. Some studies use different approaches to partitioning data such as parametric stochastic modelling approach Andersen et al. (2002); Chernova et al. (2003); Aït-Sahalia (2004); Andersen et al. (2007) without referring to Shewhart principles. These models were mainly developed for volatility forecasting. It is well known, however, that the jump components are significantly less predictable than the continuous path components, and that partitioning therefore leads to different roles in the forecasting context. Barndorff-Nielsen and Shephard (2004) used the sums of powers (realized power variation) and products of powers of absolute returns (bi-power variation) and the difference between these two measures as the quadratic variation of the jump component. The more recent methodology of Barndorff-Nielsen and Shephard (2001, 2002) as well as the ARCH (Engle, 1982) and GARCH (Bollerslev, 1987) modelling and their variants do not provide a way of extracting the 'rational' component of volatility from the observed total volatility. The event study framework has also been used to discuss stock price performance for abnormal stock returns, see Brown and Warner (1980); Kothari and Warner (1997); Yu and Leistikow (2011). The quality control procedure suggested here brings a wider range of useful information to the investors than focusing on abnormal events or returns.

In Section 9.2, standard deviation (measured volatility) of an individual stocks is used to compare performance. Sections 9.3 and 9.4 propose two risk measures based on the observed variation in special and common cause periods of stock returns for short term and long term investors respectively. It is well known that risk averse long term investors wish to maximize their return for any given level of risk or, put another way, minimize their risk for any given level of return. On the other hand, short term traders typically seek out more volatile instruments to trade. These traders tend to apply the adage of "cutting losses while letting profits run" so instruments with a lot of price variation give them the most opportunity to profit. It makes little sense for long term investors to concern themselves with short term price movements and volatility but these price movements are of critical importance to short term traders.

9.2 Comparing performances of stocks using the new measures

The first scenario uses forty stocks randomly selected from the S&P 500 list for different sectors to demonstrate the use of the new measures. The standard deviation can be used as a possible risk measure for stock returns, see Sharpe (1964). The results for this sample are presented in Table 9.1: $\hat{\sigma}_C$ is the common cause variation estimate based on daily data, while $\hat{\sigma}_C$ rank is a stock's ranking based on its common cause variation estimate relative to the other stocks, with a rank of 1 being lowest and a rank of 40 being highest. $\hat{\sigma}_L$ is the total variation estimate (standard deviation) based on daily data and the $\hat{\sigma}_L$ rank column contains each stock's rank based on this variable. The $(\hat{\sigma}_L - \hat{\sigma}_C)$ column contains the special cause variation estimate for each stock and the ranks are in the adjacent column. Finally the total variation estimate based on weekly returns data is presented in column $\hat{\sigma}_L^{\text{weekly}}$ and each stock's ranking based on this measure is in the adjacent column.

The estimate of common cause variation is likely to be more relevant for short term traders. It is well known that long term investors should not concern themselves with short term price movements, so it is common for this category of market participants to focus on lower frequency data. For instance, they may limit their analysis to weekly data rather than pouring over daily data. Estimates generated from weekly data to common cause and special cause variation estimated from weekly subgroups of daily data are therefore compared and these two techniques result in quite different stock rankings.

For long term investors, a risk measure based on the special cause variation is more relevant than total variation. Those who wish to choose the most volatile of these stocks to trade would make a similar selection regardless of whether they used common cause variation or total variation to make their choice. Based on common cause variation estimates they would conclude that J.C. Penney (followed by Petr) is the most volatile of these stocks. These stocks are also given a high rank for total variation. The least attractive stock for the noise-trader would be Johnson & Johnson on the basis of common cause variation, which is also given the lowest rank for total variation.

The Bank of America Corp stock shows a low total variation, but we now see that this is due to their low special cause variation and in fact their common cause variation is relatively high with a rank of 32; it should therefore be quite attractive to the noise-trader. Some stocks that would be less attractive to the noise-trader (for example, Coca-Cola) due to their low rank (is 5) in terms of common cause, are also less attractive to the long term investor who wishes to avoid the risky stocks if they base their judgement on the special cause variation measure (rank is 37).

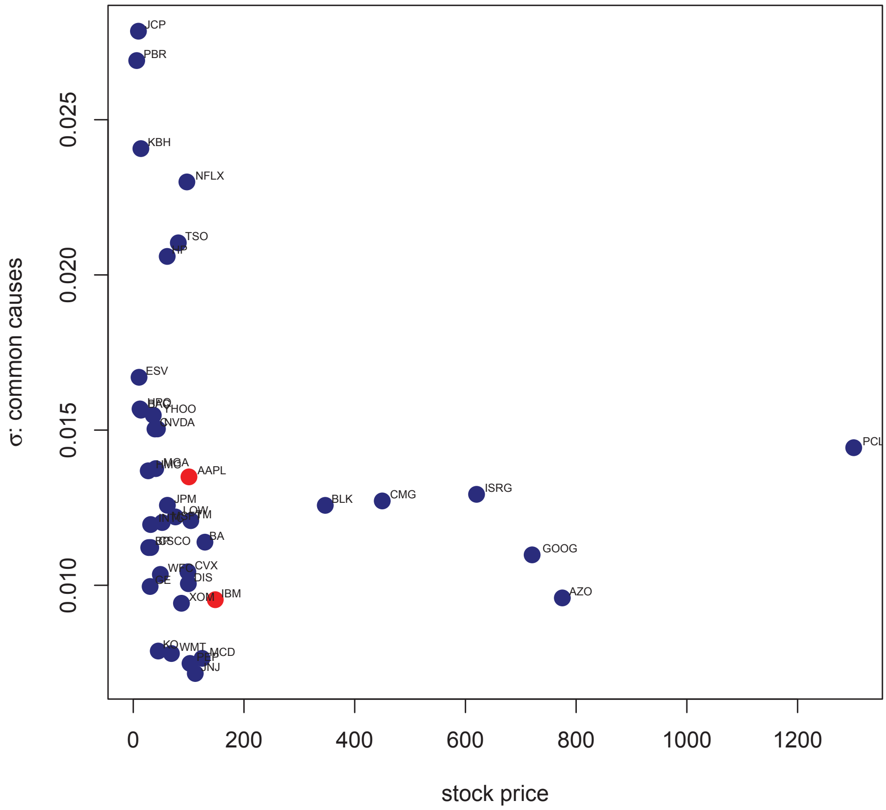
On the whole, the three measures of variation are seen to yield different orderings of least to most risky stocks. Use of weekly returns data instead of daily returns seems to support the use of a risk measure based on total variation; on the basis of these findings it is not a substitute for use of the special cause variation measure.

The analysis of a wide range of U.S. equity market data suggested that long term investors would reach quite different conclusions regarding risk if they solely focused on the estimates of volatility based on the observed total variation in a long series of past high frequency data. This is a simple approach that market participants could use as another tool to evaluate stock performances.

9.3 What do short term investors look for?

While market volatility can have a significant effect on investments over the short term, its impact tends to be smoothed out over time. Therefore, at given point in time, a short

Figure 9.1: Measured volatility (σ) in the common cause periods vs. average stock price



term trader would look at the amount of measured volatility in the context of the price to be paid for the stocks. The easiest way to present the opportunities is using a graph showing common cause volatility $\hat{\sigma}_C$ against current price and this is given in Figure 9.1 for the selected sample of stocks. Volatility measured in common cause periods during January-2012 to June-2016 and average price was calculated from the data in March-2016 to June-2016. For example IBM and Apple Inc. have nearly equal share prices, but their common cause variation has a significant difference. So, the short term trader will find better return opportunities by purchasing Apple stock.

9.4 What do long term investors look for?

In Section 3.2, the first four moments were analysed for the selected total time period. In this section, historical returns data of Apple stock from 1985 to 2015 was analysed on a yearly basis using weekly subgroups. Figure 9.2 gives the variations (standard deviation) between total and common/special cause periods over time. Figure 9.2 shows that in 2014, the total variation reached a high level but common cause variation remained stable. It is likely that this was due to a seven-for-one stock split in Apple stock. This figure also shows that standard deviation in common and special cause periods yield different results on yearly basis which is important for long term investors.

Figure 9.3 shows, the risk-return scatter plot and it is one of the best tools to illustrate risk vs. return of an investment and helps investors make decisions based on their goals and risk tolerance. The risk for long term investors is assumed to be from special cause volatility. The investors' goal may be to increase returns while decreasing risk and increase returns without increasing risk. According to our results, Hewlett-Packard and Google Inc. show approximately equal amount of risk (volatility: σ in the special cause periods) but Google gives a higher return than the Hewlett-Packard stock. So, long-term investors comparing these two stocks alone would move to the upper left portion of the figure to achieve their investment goals.

Results have been shown for a selected set of stocks in order to conserve space while technique. Portfolio managers will of course need to deal with hundreds of stocks that are openly traded across the world. Ascertaining the data given in Table 9.1 is not difficult, and standard quality control charts for the process mean and standard deviation can be generated easily within many statistical packages. The investor can then investigate the reasons behind the excess volatility in time periods which are identified by extreme points in the control chart (see Figure 3.1) for standard deviation. Use of a control chart will

Figure 9.2: Standard deviation (measured volatility) in total, common and special cause periods: Apple stock

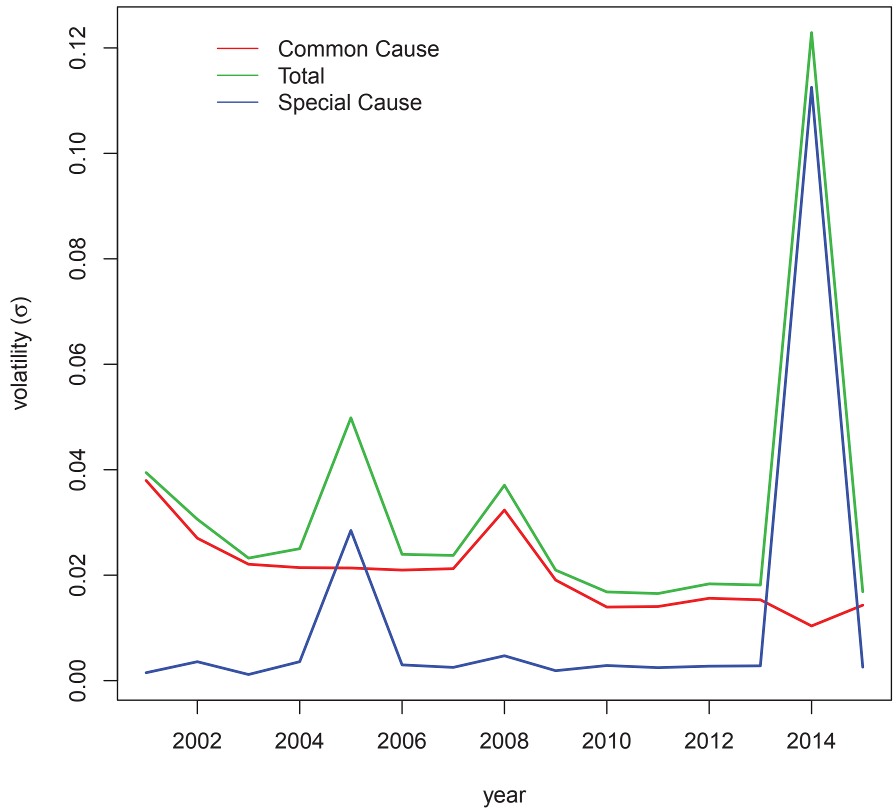
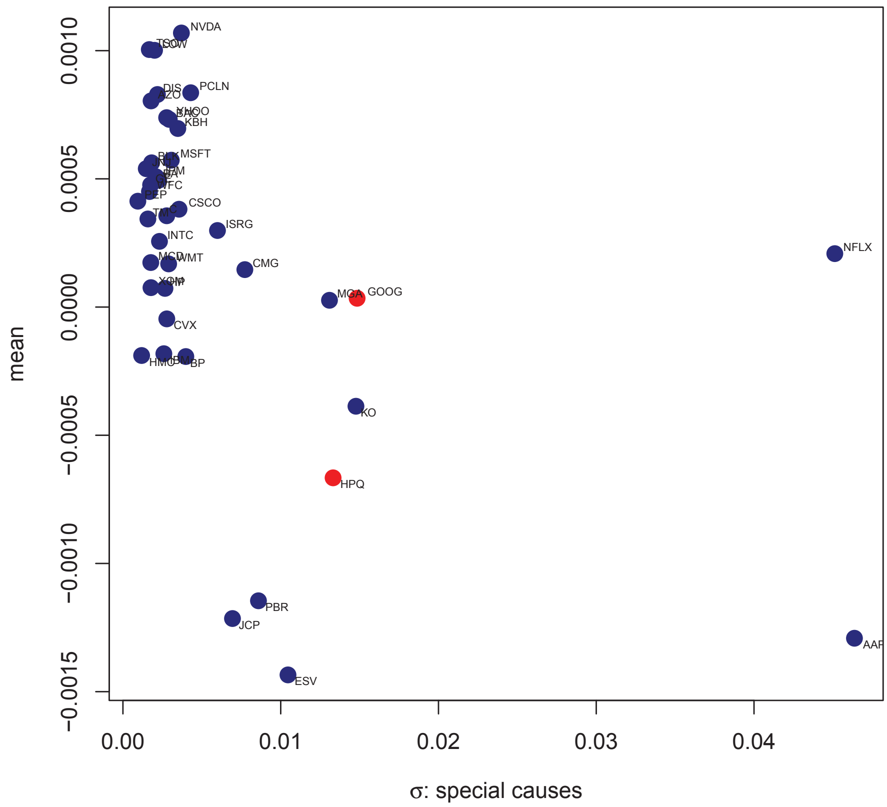


Figure 9.3: Mean average return vs. measured volatility (σ) in the special cause periods



also show the investor what volatility is common for each stock, and contribute to greater understanding of the randomness that occurs in stock price movements.

In terms of findings, it is suggested that long term investors need not concern themselves with short term noise induced volatility when assessing risk, but should rather focus on special cause volatility, whereas short term traders should focus on the volatility induced by common causes.

This approach uses daily rather than intraday data, but the methodology can be easily extended to intraday data. This study has demonstrated that the technique of rational subgrouping is a more effective way of removing noise from data than simply using lower frequency data. In other words, this work suggests that using a full data set (e.g. daily data) and then drawing inferences based on a technique most appropriate to an individual is a superior approach to selecting lower frequency data (e.g. weekly data). Even if these measures are not used in formulating portfolios and other investment decisions, they do expose elements of the stock return distributions that need to be given more thorough inspection.

Table 9.1: Standard deviation (σ) in common cause, special cause, and total data and stock rankings

Company	$\hat{\sigma}_C$	$\hat{\sigma}_C$ rank	$\hat{\sigma}_L$	$\hat{\sigma}_L$ rank	$(\hat{\sigma}_L - \hat{\sigma}_C)$	$(\hat{\sigma}_L - \hat{\sigma}_C)$ rank	$\hat{\sigma}_L^{\text{Weekly}}$	$\hat{\sigma}_L^{\text{Weekly}}$ rank
Apple Inc.	0.013498	25	0.059864	39	0.046367	40	0.026772	39
AutoZone Inc	0.009598	8	0.011381	6	0.001783	10	0.004883	6
Bank of America Corp	0.015646	32	0.018584	24	0.002937	23	0.008565	25
BlackRock	0.012580	21	0.014398	16	0.001818	11	0.006176	15
Boeing Company	0.011394	16	0.013658	12	0.002264	15	0.006144	14
BP plc	0.011222	15	0.015208	21	0.003987	28	0.006622	19
Chevron Corp.	0.010438	12	0.013219	11	0.002781	21	0.005905	12
Chipotle Mexican Grill	0.012722	23	0.020451	28	0.007728	32	0.008931	26
Cisco Systems	0.011220	14	0.014775	18	0.003555	26	0.006755	20
Citigroup Inc.	0.015043	30	0.017819	22	0.002776	20	0.007760	22
The Coca Cola Company	0.007886	5	0.022657	29	0.014770	37	0.010029	29
Ensco plc	0.016707	34	0.027169	34	0.010462	34	0.012458	34
Exxon Mobil Corp.	0.009425	6	0.011202	5	0.001777	9	0.004831	4
General Electric	0.009966	9	0.011720	7	0.001754	7	0.005391	8
Google Inc.	0.010985	13	0.025828	32	0.014843	38	0.012070	32
Helmerich & Payne	0.020601	35	0.023266	31	0.002665	18	0.010340	31
Hewlett-Packard	0.015687	33	0.029009	36	0.013321	36	0.012662	36
Honda Motor	0.013693	26	0.014878	19	0.001185	2	0.006310	17
International Bus. Machines	0.009540	7	0.012134	9	0.002594	17	0.005592	10
Intel Corp.	0.011962	17	0.014280	15	0.002318	16	0.006205	16
Intuitive Surgical Inc.	0.012935	24	0.018928	27	0.005993	30	0.008186	24
J.C.Penney	0.027857	40	0.034801	37	0.006944	31	0.017472	38
Johnson & Johnson	0.007162	1	0.008641	2	0.001479	3	0.003747	2
JPMorgan Chase & Co.	0.012582	22	0.014680	17	0.002098	13	0.006423	18
KB Home	0.024074	38	0.027554	35	0.003480	25	0.012471	35
Lowe's Cos.	0.012208	20	0.014212	14	0.002004	12	0.006051	13
Magna Inter.	0.013764	27	0.026857	33	0.013093	35	0.012163	33
McDonald's Corp.	0.007650	3	0.009416	3	0.001765	8	0.004040	3
Microsoft	0.012026	18	0.015091	20	0.003065	24	0.006788	21
NetFlix Inc.	0.023000	37	0.068122	40	0.045122	39	0.029527	40
Nvidia Corporation	0.015040	29	0.018745	26	0.003705	27	0.008946	28
PepsiCo Inc.	0.007487	2	0.008433	1	0.000946	1	0.003679	1
Petr	0.026911	39	0.035502	38	0.008591	33	0.015724	37
Priceline.com Inc	0.014438	28	0.018732	25	0.004295	29	0.008944	27
Tesoro Petroleum Co.	0.021038	36	0.022709	30	0.001671	5	0.010096	30
Toyota Motor	0.012088	19	0.013671	13	0.001583	4	0.005648	11
Wal-Mart Stores	0.007805	4	0.010704	4	0.002898	22	0.005008	7
The Walt Disney Company	0.010056	10	0.012245	10	0.002189	14	0.005559	9
Wells Fargo	0.010354	11	0.012037	8	0.001682	6	0.004841	5
Yahoo Inc.	0.015484	31	0.018253	23	0.002769	19	0.007861	23

9.5 Summary

Long term investors seek to minimize their risk for any given level of return, but short term traders typically seek out more volatile instruments to trade. Long term investors do not need to concern themselves with common cause related observed variation but this is important to short term traders. This chapter investigated how the principle of rational subgrouping can be applied to financial data to generate more appropriate volatility estimates for short and long term investors. This chapter also showed how investors with a longer term perspective would have a different view of risk if they focused on special cause variation, which is more appropriate for them, rather than total variation.

Chapter 10

Concluding remarks

This study has comprehensively investigated the use of Shewhart's postulates on common and special cause variations to model stock returns. A set of new propositions based on the Shewhart philosophical view were empirically examined for partitioned financial data. New statistical methods and software relevant to financial data were presented. Several important research issues were also identified in this study relating to the distribution of stock returns, asset pricing and portfolio management. This chapter summarises the study and future opportunities are described.

10.1 Summary of objectives and achievements

As mentioned above, the appropriateness of using Shewhart's principles for financial data has been comprehensively discussed in the preceding chapters. Three new propositions were investigated, and contentious financial issues were addressed using the first four moments, co-moments and autocorrelation in the distribution of stock returns. The following three objectives were identified in the beginning of this study and this section summarises how and to what extent these objectives have been achieved.

- Investigate the appropriateness of the Shewhart approach for assessing the behaviour of stock returns.
- Demonstrate that this approach will help to understand the contradictory arguments and empirical findings published in the financial literature.
- Show that Shewhart's principles can be used to understand the behaviour of stock returns.

Chapter 2 laid the foundation for using Shewhart's philosophy to understand the source of variations in stock returns. Different contexts were chosen from financial market practices and the activities in a production process inter alia for the validation of the Shewhart philosophy. Shewhart's notion of common and special causes was compared with rational and irrational market behaviour. Disparities encountered in the application of Shewhart principles to financial markets and compared to industrial processes were discussed. Chapter 2 has substantially contributed to the study's first objective as listed above.

In Chapter 3, all the techniques that have been used to partition common and special cause periods in stock returns were listed. Partitioning data is essential for investigating the three propositions and addressing the issues from the application of Shewhart principles; therefore, statistical methods were identified and modified to accommodate the nature of financial data. Robust estimators were considered for unknown parameters such as μ and σ. A robust chart procedure based on these robust estimators was used to separate common and special cause periods. Throughout this study, all the propositions that were put forward in Section 1.2 were tested empirically using S&P 500 stocks and the S&P 500 index itself. The partitioning procedures and the effect of subgroup size selection were demonstrated using selected stocks. For this study, the first four moments in stock returns are crucial; therefore, the partitioned data was examined using them. It was found that excess skewness and kurtosis become highly significant in special cause periods for most of the stocks examined. These results were statistically tested in Chapter 4. The techniques that have been considered in this study are not limited to financial data but can be extended to other time series data. The QCCTS (Quality Control Charts for Time Series)-R package was developed for this purpose.

In Chapter 1, the following three propositions were put forward.

Proposition 1: The distribution of stock returns will follow a normal distribution under common cause variations.

Proposition 2: The joint distribution of an asset return and market return will follow a bivariate normal distribution under common cause variations.

Proposition 3: Stock returns will show zero autocorrelation in the common cause periods.

The second objective for the study was to address some of the unresolved research questions found in the empirical finance literature. In Section 1.1, the following three issues were discussed in detail:

- Is there always a positive risk–return trade–off?
- Are co-moments measured accurately within extensions to the two-moment capital asset pricing model?
- How does autocorrelation in stock returns affect market efficiency?

Chapters 4, 6 and 8 have addressed the above three propositions and research issues whilst Chapters 5 and 7 have described the new statistical techniques.

To address the risk/return trade–off issue, partitioned data from S&P 500 stocks were examined in Chapter 4. Investigating into the first four moments in the total, common and special cause periods, excess skewness and excess kurtosis were both found in the total data but not in common cause periods, *Proposition* 1 has therefore been justified. Mean–standard deviation and skewness–kurtosis trade–offs were first investigated from known theoretical distributions and then empirical data was examined. Significant positive and negative mean–standard deviation trade–offs were found after partitioning the returns series into common and special cause periods respectively. A highly negative skewness–kurtosis trade–off was found in total and special cause periods as compared to the common cause periods. This analysis demonstrated that partitioning of the data in this way provides a useful resolution to the risk–return trade–off issue (*Issue* 1) documented in the literature.

Proposition 2 was based on co-moments in return distributions. In order to investigate the co-moments, a bivariate control chart is required to determine the common and special cause variations. Market and stock returns are available as paired data, therefore in Chapter 5, difference control charts were investigated for monitoring the paired data. Shewhart charts for the mean of the difference series have been proposed in the literature but their performance has not been investigated. Hence, the run length properties of the difference charts for mean and standard deviations were examined and compared with other well-known bivariate charts when parameters are known and estimated from a small sample of Phase I data. Moreover, the inverse-Wishart (IW) prior distribution was used to generate the unknown shifts in the process for further evaluation of the charts. Difference charts performed better than the existing charts. They need not be limited to this study but could be used in other applications of paired data. In Chapter 6, the difference series was used to determine common and special causes for bivariate stock returns.

The second research issue on capital asset pricing models was addressed using the co-skewness and co-kurtosis in the total and common cause periods (Chapter 6). The impact of one-off market events was identified in the total and common cause periods and thereby justifying *Proposition* 2. This analysis therefore provides additional information relevant for asset pricing. A new set of estimates for co-moments which does not require the knowledge of the true correlation coefficient is proposed. New measures were based on the differences in all the non-matching pairs of two variables. Using Monte Carlo simulation, it was confirmed that the new measures performed equally or better than the existing bivariate co-moments. To improve the efficiency, using a sub-sample of non-matching pairs when the sample size is large was suggested. The partitioning of data helps to understand the role of co-moments and their impact on asset pricing models (*Issue* 2).

The third research issue refers to autocorrelation in stock returns. In Chapter 7, a new approach was proposed. The odd–even split reduces the effect of underlying autocorrelation. Using known time series models, this study has derived autocorrelation functions for odd-even series. The contribution made in this chapter is not limited to this study but could be transferred to other applications.

Using the technique proposed in Chapter 7, autocorrelation in actual stock returns was examined in Chapter 8. Using a sufficient number of examples, the nature of the induced autocorrelation generated by the definition of log returns was explained. These results were then used to analyse autocorrelation in returns of stock in the S&P 500 list. In a large proportion of stocks, inconsistent autocorrelation functions in the odd and even series were shown to be caused by the outliers in the complete return series. Three trial methods were used to reduce the effect of outliers, and less discordance between the autocorrelation functions in odd and even series results when trimming method was used. The impact of other factors including day effect, initial value of the return series, trading volumes and market capitalization and their effect on autocorrelation was examined and results showed that those factors were not significant. A good number of stocks were found to exhibit non-zero autocorrelation in the common cause periods (*Proposition*

3). By showing the effect of unusual observations on patterns of autocorrelation and the induced autocorrelation by definition of stock returns, this chapter have discussed the *Issue* 3.

Chapter 9 has discussed some examples of how investors can estimate risk based on the moments in partitioned data. Potential applications have been identified and will be briefly mentioned later in this Chapter.

The three propositions were largely justified using the partitioned data, therefore the first objective has been completely achieved. This study has demonstrated that using Shewhart principles can clarify some of the issues which were unresolved in the literature therefore the second objective has been achieved. The benefits of viewing those issues from a different perspective were displayed. Sufficient evidence has been presented to show that the use of Shewhart principles to understand the behaviour of financial return series may lead to the provision of more accurate and detailed advice to market participants, thus achieving third objective.

10.2 Future research directions

The following suggestions are made for advancing the current study. In Chapter 3 a parametric approach was used to partition common and special cause periods. A recent publication Delaigle and Hall (2016) proposes a probabilistic approach of non-parametric deconvolution methods, therefore the partition procedure might be improved using this approach. On the other hand, Exponentially Weighted Moving Average (EWMA) or Cumulative Sum (CUSUM) control charts (see Chapter 9 of Montgomery (2011)) are used for monitoring the mean of auto-correlated process. However, these charts induce autocorrelation by the design and from Chapters 7, 8 autocorrelations in stock returns is a central issue. So the problem become complicated but could be considered.

Several web based data sources are available for historical stock price series and market indices. In addition to the QCCTS R-package, a web based shiny app would be useful for investors because knowledge about using a software would not required; an app may be more user friendly.

In this study, skewness and kurtosis were calculated for fixed time intervals. DeFusco et al. (1996); Adcock and Shutes (2005) have examined skewness persistence in stock returns over different time intervals. Since investors use skewness as a risk measure, this type of analysis for common cause data may provide more useful results. Lau and Wingender (1989) have considered the effect of interval of differencing log returns for skewness and kurtosis. Our study is limited to daily returns but the analysis could be re-examined for low frequency (weekly, monthly) or high frequency (hourly) time intervals of data. Also without assuming the sample distribution of sample skewness or kurtosis, one can use a bootstrapping approach to test the significance of excess skewness and kurtosis.

Run length properties of the $\bar{d}$ and S_d charts were discussed for the one point signal rule. Further research is required to assess the performance of these difference control charts for other signal rules and the effect that exponential smoothing and cumulative sums on the differences may have.

In Chapter 6, new measures based on the differences of non-matching pairs of two series were proposed. To improve the computational efficiency of these estimators, a sub-sample of non-matching pairs was suggested. The false alarm rates for the new measures estimated using sub-samples can be further examined.

Further investigation is also needed to examine and interpret ACF/PACF in the complete series from the odd–even split when the time series model for the complete series is unknown. In Chapters 7 and 8, the use of the odd–even split was limited to stock returns but it can be examined for other types of time series data.

A real-time portfolio analysis based on partitioned data also lay outside the scope of this study, but would provide further confirmation of the method's validity and practicality. This study has provided the tools for a CAPM investigation of partitioned data using both QCCTS and PortfolioAnalytics–R (Peterson and Carl, 2015) packages. Possible applications in different types of financial series and other disciplines were noted briefly.

In addition, Chapter 9 showed a few applications from an individual investors' point of view. So the findings and techniques of this thesis can be further investigated for activities of several investors using an agent simulation model to examine the aggregate change in the market. An agent simulation model is an artificial model for financial market that investors activities can be artificially simulated created, see Marco et al. (2001); Blake (2000). These approaches are worthy of future research for validation purposes.

10.2.1 Further applications in finance

The preferences of investors for the use of skewness and kurtosis as risk measures has been discussed by many authors including Scott and Horvath (1980); Baillie and DeGennaro (1990); Fang and Lai (1997); Kim and White (2004). Even after removing the periods with excess volatility in returns series, significant skewness and kurtosis were found in the returns during common cause periods (Section 4.2). According to Cornell (2009), some investors prefer high volatility and skewness during crisis periods, therefore there will be potential advantages by comparing skewness and kurtosis in partitioned data.

Co-moments have also had similar attention from investors using asset pricing models. Chapter 6 has shown that the effect of infrequent market events on co-moment behaviour by comparing common and special cause variations. A family of risk measures which are based on co-moments can be found in Chen (2016, p.212) thus investigating them using partitioned data may provide additional information to investors. Further research is required to demonstrate how these results can be analysed from an investor's point of view by considering actual portfolios.

In today's world, there are many types of financial instruments in addition to stocks. The most common examples include options, indices, future contracts, bonds, currencies and swaps and some of them are designed based on the stocks. In this study, Shewhart principles were investigated for stock returns, but other types of financial series might need a similar analysis because all financial series are largely affected by unforeseen market events. Market participants experience different sources of variations depending on many factors including the type of financial security, position on the contract, size of the portfolio and market changes. Forecasting or measuring volatility is therefore an essential task for any type of financial series: for example Aït-Sahalia et al. (2001) examined the effect of volatility in pricing options; Chen et al. (2006) investigated volatility persistence in future contracts by considering different measures; and Demos and Goodhart (1996) examined the behaviour of the foreign exchange (FOREX) market in terms of volatility and some other factors.

In many scenarios, the first four moments and co-moments have been considered: Conrad et al. (2013) explored relationships between the first four moments in option prices; Baker et al. (2017) found a strong correlation between the first four moments in monthly returns of a hedge fund; Ranaldo and Favre (2005a) considered co-moments to assess hedge fund performances and concluded that the two-moment or multivariate-moment model should be used according to the risk/return trade–off; Christie-David and Chaudhry (2001) showed the increased explanatory power by inclusion of co-moments for future contracts; and Liow and Chan (2005) tested the four-moment CAPM in real estate securities.

The Shewhart approach, therefore, is a valuable way of analysing the behaviour of these financial securities but the empirical results are expected to differ depending on the type of the financial instrument. The direct use of difference control charts has also been noted for financial series. Garthoff et al. (2014a) considered monitoring of bivariate series of daily FOREX log returns to detect possible mean changes; Mitchell et al. (2006) noted that all sectors are not traded equally and that information dissemination is different across the sectors. The difference control charts are therefore particularly suitable for this application. The scenarios presented above demonstrate that research in empirical finance would benefit using methods based on the Shewhart philosophical view of variation.

10.2.2 Applications in other disciplines

Applications can be found in other contexts that could benefit from the use of some of the methods presented in this study. It is suggested, for example, the odd-even split approach is useful for applications such as Markov chain Monte Carlo (MCMC) simulations. In Bayesian literature, thinning of chains in MCMC simulations is sometimes done so that the effect of autocorrelation is reduced (see Link and Eaton, 2012). The odd-even split approach can be extended to a four or multiple ways of partitioning a complete series; therefore, it is expected that a multiple partitioning approach will be useful for Bayesian estimation using MCMC samples.

Jensen et al. (2006) suggested that it is desirable to first remove the impact of autocorrelation before using data for control charts. Woodall and Montgomery (2014) noted the issues of fitting a time series model or the assumption of a known time series model when monitoring autocorrelated data. The split series approach may be a better solution to solve this issue. In this case, there is an opportunity for further research into control charts of split series and the interpretation of the special causes.

Bivariate paired data appears in several situations in medical studies and industrial applications such as survival rate analysis in (Huster et al., 1989, p.149), (Kim, 1995, p.1344), and measurements comparisons in machines Grubbs (1946). The use of difference control charts is therefore relevant to these applications.

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Terms, definitions and notations

IQR_i	interquartile range of subgroup i, where $Q_{i,1} = x_{i,(a)}$ and $Q_{i,3} = x_{i,(b)}$, with $a = \lceil n_i/4 \rceil$, $b = n - a + 1$, and $x_{i,(m)}$ the m^{th}–order statistics of subgroup i, (Nazir et al., 2014b, p.132). 36
N	length of the series. 29
$S^2_{d_i} = \frac{1}{n-1} \sum_{j=1}^{n} \left(d_{ij} - \bar{d}_i\right)^2$	Subgroup standard deviation of the difference series. 66
$X_{ij}, i = 1,2,\ldots,m \;\&\; j = 1,2,\ldots,n_i$	data series. 29
$\bar{S}_\alpha = \frac{1}{m-2\lceil m\alpha \rceil} \left[\sum_{i=\lceil m\alpha \rceil+1}^{m-\lceil k\alpha \rceil} \bar{s}_{(i)}\right]$	trimmed mean of subgroup standard deviation, where α denotes the percentage of subgroup data to be trimmed, $\lceil v \rceil$ denotes the ceiling function and $\bar{s}_i$ denotes the ith subgroup sample standard deviation. 30
$\bar{\bar{d}} \pm 3 \frac{\bar{S}_d}{c_4(n)\sqrt{n}}$	the three-sigma upper control limit (UCL) and lower control limit (LCL) when the parameters are unknown. 66
$\bar{d}_i = \frac{1}{n} \sum_{j=1}^{n} d_{ij}$	Subgroup means of the difference series. 66

$\frac{\bar{S}_d}{c_4(n)}\sqrt{\frac{\chi^2_{0.00135}}{n-1}}$ LCL for S_d chart, where $\bar{\bar{d}}$ is the average subgroup mean of the differences and $\bar{S}_d$ is the average subgroup standard deviation of the differences. 66

$\frac{\bar{S}_d}{c_4(n)}\sqrt{\frac{\chi^2_{0.99865}}{n-1}}$ UCL for S_d chart, where $\bar{\bar{d}}$ is the average subgroup mean of the differences and $\bar{S}_d$ is the average subgroup standard deviation of the differences. 66

$\gamma_k = \text{Cov}[x_t, x_{t-k}]$ autocovariance at lag k of the complete series. 119

$\gamma_{k,\text{e}}$ autocovariance at lag k of the even series. 119

$\gamma_{k,\text{o}}$ autocovariance at lag k of the odd series. 119

$\text{Corr}(x_{t,\text{o}}[t+k], x_{t,\text{e}}[t])$ two-sided cross-correlation between odd and even series at lag k. 119

$\text{Corr}(x_{t,\text{o}}[t], x_{t,\text{e}}[t])$ two-sided zero lag cross-correlation between odd and even series. 119

$\mu_d \pm 3\frac{\sigma_d}{\sqrt{n}}$ the three-sigma upper control limit (UCL) and lower control limit (LCL) for the $\bar{d}$ chart, $\mu_d = \mu_x - \mu_y$ and $\sigma_d^2 = \sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y$. 66

$\overline{IQR}_{10} = \frac{1}{k-2(\lceil k/10\rceil-1)}\left[\sum_{m=\lceil k/10\rceil}^{k-\lceil k/10\rceil+1} IQR_{(m)}\right]$ 10% trimmed mean of the sample interquartile ranges, Nazir et al. (2014b, p.132). 36

$\overline{TM}_{10} = \frac{1}{k-2\lceil k/10\rceil}\left[\sum_{m=\lceil k/10\rceil+1}^{k-\lceil k/10\rceil} TM_{(m)}\right]$ 10% trimmed mean of the sample trimeans, Nazir et al. (2014a, p.248). 38

$\phi_{kk,\text{e}}$ partial autocorrelation at lag k of the even series. 119

$\phi_{kk,\mathrm{o}}$	partial autocorrelation at lag k of the odd series. 119
ϕ_{kk}	partial autocorrelation function of the complete series. 119
$\rho_k = \frac{\gamma_k}{\gamma_0}$	autocorrelation at lag k of the complete series. 119
$\rho_{k,\mathrm{e}}$	autocorrelation at lag k of the even series. 119
$\rho_{k,\mathrm{o}}$	autocorrelation at lag k of the odd series. 119
$\sqrt{\frac{\chi^2_{n-1;0.00135}}{n-1}}\sigma_d$	LCL for S_d chart, where $\sigma_d^2 = \sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y$. 66
$\sqrt{\frac{\chi^2_{n-1;0.99865}}{n-1}}\sigma_d$	UCL for S_d chart, where $\sigma_d^2 = \sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y$. 66
$\widehat{S}_c = \frac{N}{(N-1)^{1/2}} \frac{\left(\sum_{\lvert u_i\rvert<1}(x_i-T)^2(1-u_i^2)^4\right)^{1/2}}{\left\lvert\sum_{\lvert u_i\rvert<1}(1-u_i^2)(1-5u_i^2)\right\rvert}$	"Biweight A" estimator of σ, where T is the sample median, $u_i = \frac{(x_i-T)}{(c\mathrm{MAD})}$, $d_i = \lvert x_i - T\rvert$, $\mathrm{MAD} = \mathrm{med}\{d_1, d_2, \cdots d_n\}$ (Lax, 1985, p.739). 39
$\{x_t;\ t = 1, \ldots, n\}$	a complete series. 118, 119
$d = X - Y$	Difference series of (X, Y) two correlated variables. 65
m	number of subgroups. 29
n_i	subgroup size. 29
r_k	sample autocorrelation function at lag k of the complete series. 119
$r_{k,\mathrm{e}}$	sample autocorrelation at lag k of the even series. 119

$r_{k,\mathrm{o}}$	sample autocorrelation at lag k of the odd series. 119
$r_{kk,\mathrm{e}}$	sample partial autocorrelation at lag k of the even series. 120
$r_{kk,\mathrm{o}}$	sample partial autocorrelation at lag k of the odd series. 120
r_{kk}	sample autocorrelation function at lag k of the complete series. 120
$s_i = \sqrt{\frac{1}{n_i-1}\sum_{j=1}^{n_i}\left(X_{ij}-\bar{X}_i\right)^2}$	subgroup standard deviation. 29
$x_{t,\mathrm{e}} : \{x_{2t};\ t = 1,\ldots,\frac{n}{2}\}$	even series. 118, 119
$x_{t,\mathrm{o}} : \{x_{2t-1};\ t = 1,\ldots,\frac{n}{2}\}$	odd series. 118, 119
aleatory	The uncertainty in this case attributed to the physical world because human failed to reduce or eliminate it by enhancing the underlying knowledge base,(Ayyub, 2014, p.38). 65
ARMA-GARCH model	$Y_t = \mu_0 + \sum_{i=1}^{p}\phi_i Y_{t-i} + \epsilon_t - \sum_{j=1}^{q}\theta_j\epsilon_{t-j}, \epsilon_t = \sqrt{h_t\eta_t}, h_t = \omega_0 + \sum_{i=1}^{Q}\alpha_j\epsilon_{t-j}^2 + \sum_{j=1}^{P}\beta_j h_{t-j}$. 142
autocorrelation function	$\rho_k = \frac{\mathrm{E}[(r_t-\mu)(r_{t+k}-\mu)]}{\sqrt{\mathrm{E}\left[(r_t-\mu)^2\right]\mathrm{E}\left[(r_{t+k}-\mu)^2\right]}}$. 137
autoregressive model of order p	AR(p): $x_t = \mu + \sum_{i=1}^{p}\phi_i x_{t-i} + \epsilon_t$, where $\epsilon_t \sim N(0,\sigma_\epsilon^2)$. 120
autoregressive moving average model	ARMA(p,q): $x_t = \mu + \sum_{i=1}^{p}\phi_i x_{t-i} + \epsilon_t + \sum_{j=1}^{q}\theta_j\epsilon_{t-j}$, where $\epsilon_t \sim N(0,\sigma_\epsilon^2)$. 127

average run length	The average number of points that must be plotted before a point indicates an out-of-control condition,(Montgomery, 2011, p.191). 64
bear market	A market distinguished by declining prices, Archer (2012, p.176). 44
bid-ask bounce	Successions of buy and sell orders take place at different prices, even if the price of the asset is constant, (Bauwens and Giot, 2013, p.41). 5
bid-ask spread	A bid-ask spread is the amount by which the ask price exceeds the bid price for an asset in the market, Campbell et al. (1997, p.99). 5
bull market	A market distinguished by rising prices, Archer (2012, p.176). 44
capital asset pricing model	A model based on the proposition that any stock's required rate of return is equal to the risk-free rate of return plus a risk premium reflecting only the risk remaining after diversification, Brigham and Daves (2014, p.1123). 3
diffuse disturbances	Outliers that are spread over all of the samples; (Schoonhoven et al., 2011, p.365). 28

epistemic	The uncertainty present as a result of a lack of or deficiency in knowledge ,(Ayyub, 2014, p.38). 65
inverse-Wishart (IW)	$\Sigma \sim IW(\nu, \Lambda)$,inverse Wishart (IW) distribution, where Λ is a positive d dimensional matrix and ν represents the degrees of freedom.. 80
moving average model of order q	MA(q): $x_t = \mu + \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j}$, where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. 125
non-synchronous security trading	The nonsynchronous trading or nontrading effect arises when time series, usually asset prices, are taken to be recorded at time intervals of one length when in fact they are recorded at time intervals of other, possibly irregular,lengths, Campbell et al. (1997, p.84). 4
partial price adjustments	Trade takes place at prices that do not fully reflect the traders' information, Anderson et al. (2013, p.79). 5

portfolio	A group of individual assets held in combination. An asset that would be relatively risky if held in isolation may have little or no risk if held in a well-diversified portfolio, Brigham and Daves (2014, p.1140). 2
sample kurtosis	$\widehat{\text{Kurt}}_k = \frac{\frac{1}{N}\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(X_{ij}-\bar{\bar{X}}_k\right)^4}{\left(\frac{1}{N}\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(X_{ij}-\bar{\bar{X}}_k\right)^2\right)^2}$. 48
sample skewness	$\widehat{\text{Skew}}_k = \frac{\frac{1}{N}\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(X_{ij}-\bar{\bar{X}}_k\right)^3}{\left(\frac{1}{N}\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(X_{ij}-\bar{\bar{X}}_k\right)^2\right)^{3/2}}$. 47
stock returns	$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$. 2, 137
trade–off	Giving up or accepting one advantage for another to gain more value to the decision maker, Siebels (2004, p.250). 3, 44
unbiasing coefficient B_4	$b_4 = 1 + \frac{3}{c_4\sqrt{2(n-1)}}$, (Montgomery, 2011, p.702). 30
unbiasing coefficient $c_4(n_i)$	$c_4 \cong \frac{4(n-1)}{4n-3}$, (Montgomery, 2011, p.702). 30
volatility	A statistical measure of a market's price movements over time characterized by deviations from a predetermined central value (usually the arithmetic mean), Archer (2012, p.176). 3
white noise (WN) model	$\mathrm{E}[x_t] = 0, \mathrm{Var}[\epsilon_t] = \sigma_\epsilon^2$. 125

Appendix A

Statement of contribution to doctoral thesis containing publications

DRC 16

MASSEY UNIVERSITY
GRADUATE RESEARCH SCHOOL

STATEMENT OF CONTRIBUTION
TO DOCTORAL THESIS CONTAINING PUBLICATIONS

(To appear at the end of each thesis chapter/section/appendix submitted as an article/paper or collected as an appendix at the end of the thesis)

We, the candidate and the candidate's Principal Supervisor, certify that all co-authors have consented to their work being included in the thesis and they have accepted the candidate's contribution as indicated below in the *Statement of Originality*.

Name of Candidate: Liyana Pathiranaralalage Nadeeka Dilrukshi Premarathna

Name/Title of Principal Supervisor: A. Jonathan R. Godfrey, Senior Lecturer

Name of Published Research Output and full reference:

Nadeeka Premarathna , A. Jonathan R. Godfrey, K. Govindaraju (2016),"Decomposition of stock market trade-offs using Shewhart methodology", International Journal of Quality & Reliability Management, Vol. 33 Iss 9 pp. 1311 - 1331

In which Chapter is the Published Work: Chapter 4

Please indicate either:

- The percentage of the Published Work that was contributed by the candidate:

  and / or

- Describe the contribution that the candidate has made to the Published Work:

  Jonathan Godfrey and K. Govindaraju gave suggestions for research direction and guidance on how to prepare the manuscript. They both edited, proof-read the manuscript and assisted in addressing reviewer comments. Jonathan Godfrey made an additional contribution by being the corresponding author of this manuscript to demonstrate article submission procedures. The remainder of the work was carried out by the candidate (Nadeeka Premarathna) including necessary research, statistical analysis and preparation of the manuscript.

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STATEMENT OF CONTRIBUTION
TO DOCTORAL THESIS CONTAINING PUBLICATIONS

(To appear at the end of each thesis chapter/section/appendix submitted as an article/paper or collected as an appendix at the end of the thesis)

We, the candidate and the candidate's Principal Supervisor, certify that all co-authors have consented to their work being included in the thesis and they have accepted the candidate's contribution as indicated below in the *Statement of Originality*.

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Name of Published Research Output and full reference:

Nadeeka Premarathna , A. Jonathan R. Godfrey, K. Govindaraju (2017),"Control Charts for Paired Differences: $\bar{d }$ and S_d charts", Quality and Reliability Engineering International, http://onlinelibrary.wiley.com/doi/10.1002/qre.2147/full.

In which Chapter is the Published Work: Chapter 5

Please indicate either:

- The percentage of the Published Work that was contributed by the candidate:

  and / or

- Describe the contribution that the candidate has made to the Published Work:

  Jonathan Godfrey and K. Govindaraju gave suggestions for research direction and guidance on how to prepare the manuscript. They both edited, proof-read the manuscript and assisted in addressing reviewer comments. The remainder of the work was carried out by the candidate (Nadeeka Premarathna) including necessary research, statistical analysis and preparation of the manuscript.

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STATEMENT OF CONTRIBUTION
TO DOCTORAL THESIS CONTAINING PUBLICATIONS

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We, the candidate and the candidate's Principal Supervisor, certify that all co-authors have consented to their work being included in the thesis and they have accepted the candidate's contribution as indicated below in the *Statement of Originality*.

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Name of Published Research Output and full reference:

Nadeeka Premarathna , A. Jonathan R. Godfrey, K. Govindaraju (2016)," Assessment of Induced Autocorrelation in Stock Returns", Empirical Economics, Under review.

In which Chapter is the Published Work: Chapter 8

Please indicate either:

- The percentage of the Published Work that was contributed by the candidate:

  and / or

- Describe the contribution that the candidate has made to the Published Work:

  Jonathan Godfrey and K. Govindaraju gave suggestions for research direction and guidance on how to prepare the manuscript. They both edited, proof-read the manuscript and assisted in addressing reviewer comments. The remainder of the work was carried out by the candidate (Nadeeka Premarathna) including necessary research, statistical analysis and preparation of the manuscript.

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26 April 2017

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