Ultra-Cold Bosons in One-Dimensional Single- and Double-Well Potentials

Jake Steven Gulliksen

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Abstract

A variationally optimised basis allows an accurate description of the quantum behaviour of ultra-cold atoms, even in the strongly correlated regime. A rescaling scheme corrects discrepancies caused by using a reduced Hilbert space. This approach also allows the modelling of experimentally realizable double-well potentials, which still reveals the maximally-entangled states seen in fixed basis models. Time dynamics of these double-well systems show macroscopic tunnelling between wells for bosons with a sufficient interaction strength.

The many-body problem of interacting bosons in the highly-correlated regime is difficult. The number of basis states needed to describe this quantum system accurately quickly grows beyond computational reach. Rescaling the interaction strength proves a simple and effective method of calculating exact eigenvalues in a reduced Hilbert space.

Bosonic systems in the double-well potential are investigated next. First, how different eigen-states depend on the interaction strength is examined. The variationally optimised method has advantages over a standard fixed basis method with the ability to model experimentally viable systems and explore more strongly-correlated regimes. Secondly, tunnelling dynamics in the double well are studied, specifically for a system where all particles initially occupy a single well. Oscillations corresponding to collective tunnelling between wells are found in regimes where there are zero interactions or bosons lie in a maximally-entangled state. What governs the dynamics outside these two regimes is also considered.

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People travel to wonder
at the height of the mountains,
at the huge waves of the seas,
at the long course of the rivers,
at the vast compass of the ocean,
at the circular motion of the stars,
and yet they pass by themselves
without wondering.
-*Augustine of Hippo*

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