On the Descriptive Complexity of Satisfiability on Quantified Boolean Formulas

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Abstract

In the present thesis, we deal with the construction of non trivial formulas in higher order logic languages. In particular, we focus on using SO (Second-Order logic) and TO (Third-Order logic) to express $SATQBF_k$, and $SATQBF$ respectively. First of all, we explain the relationship between logic languages and complexity classes. Then we give formal definitions and examples for FO (First-Order), SO and HO^i $(i \geq 2)$ (Higher-Order logic). It is known that, for every $k \geq 1$, $SATQBF_k$ is a complete problem for the level Σ_k^P of PH (Polynomial-time hierarchy), and that $SATQBF$ is a complete problem for PSPACE. As the expressibility of SO is known to equal the class PH, then we know that there must be an SO formula which can express $SATQBF_k$. On the other hand, PSPACE is known to be equal in expressive power to SO with the addition of a second order transitive closure quantifier, which is widely conjectured to be strictly more expressive than SO alone. As TO includes $PSPACE$, this means that there must be a TO formula that can express $SATQBF$. Here we give first a top down explanation on the use of SO to express $SATQBF_k$. A detailed SO formula is presented. We then give a top down presentation of the sketch of a TO formula for $SATQBF$.

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