ON MATHEMATICAL MODELLING OF THE SELF-HEATING OF CELLULOSIC MATERIALS

A thesis presented in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Mathematics at Massey University, New Zealand

Robert Antony Sisson

June 1991

Massey University Library
Thesis Copyright Form

Title of thesis: On Mathematical modelling of the self-heating of cellulosic materials

(1) (a) I give permission for my thesis to be made available to readers in Massey University Library under conditions determined by the Librarian.

(b) I do not wish my thesis to be made available to readers without my written consent for ... months.

(2) (a) I agree that my thesis, or a copy, may be sent to another institution under conditions determined by the Librarian.

(b) I do not wish my thesis, or a copy, to be sent to another institution without my written consent for ... months.

(3) (a) I agree that my thesis may be copied for Library use.

(b) I do not wish my thesis to be copied for Library use for ... months.

Signed [signature]

Date 13 June 1991

The copyright of this thesis belongs to the author. Readers must sign their name in the space below to show that they recognise this. They are asked to add their permanent address.

NAME AND ADDRESS DATE

ROBERT SISSON 13 June 1991
273 BROADWAY AVENUE
PALMERSTON NORTH 15

ABSTRACT

This thesis considers mathematical modelling of self-heating of cellulosic materials, and in particular the effects of moisture on the heating characteristics. Following an introductory chapter containing a literature review, Chapter 2 presents some preliminary results and an industrial case study. The case study, which discusses a 'dry' body self-heating on a hot surface, investigates the following questions: (i) how hot can the surface get before ignition is likely? (ii) how well does the (slab-like) body approximate to an infinite slab? and (iii) how valid is the Frank-Kamenetskii approximation for the source term? It is shown that the minimal steady state temperature profile is stable when the temperature of the hot surface is below a certain critical value, and bounds for the higher steady state profile are derived. Chapter 3 presents the thermodynamic derivation of a reaction-diffusion model for the self-heating of a moist cellulosic body, including the effects of direct chemical oxidation as well as those of a further exothermic hydrolysis reaction and the evaporation and condensation of water. The model contains three main variables: the temperature of the body, the liquid water concentration in the body, and the water vapour concentration in the body. Chapter 4 investigates the limiting case of the model equations as the thermal conductivity and diffusivity of the body become large. In particular it is shown that, in this limiting case, the model can have at least twenty-five distinct bifurcation diagrams, compared with only two for the well known model without the effects of moisture content. In Chapter 5 the maximum principle and the methods of upper and lower solutions are used to derive existence, uniqueness and multiplicity results for the steady state solutions of the spatially distributed model. Finally, in Chapter 6, existence and uniqueness results for the time dependent spatially distributed model are derived.

I would like to express my gratitude to my supervisors Professor Graeme Wake and Mr Adrian Swift for their unfailing help and enthusiasm throughout my work. Thanks also to Dr Alex McNabb and Mr Aroon Parshotam for many helpful comments, Mr Richard Rayner for his help in producing the graphics, Miss Fiona Davies for her typing of this thesis, Joanne for her constant support, and Patricia for her friendship.

CONTENTS