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**ON MATHEMATICAL MODELLING OF THE SELF-HEATING
OF CELLULOSIC MATERIALS**

A thesis presented in partial fulfilment of the requirements for the
degree of Doctor of Philosophy in Mathematics
at Massey University, New Zealand

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June 1991

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ABSTRACT

This thesis considers mathematical modelling of self-heating of cellulosic materials, and in particular the effects of moisture on the heating characteristics. Following an introductory chapter containing a literature review, Chapter 2 presents some preliminary results and an industrial case study. The case study, which discusses a 'dry' body self-heating on a hot surface, investigates the following questions: (i) how hot can the surface get before ignition is likely? (ii) how well does the (slab-like) body approximate to an infinite slab? and (iii) how valid is the Frank-Kamenetskii approximation for the source term? It is shown that the minimal steady state temperature profile is stable when the temperature of the hot surface is below a certain critical value, and bounds for the higher steady state profile are derived. Chapter 3 presents the thermodynamic derivation of a reaction-diffusion model for the self-heating of a moist cellulosic body, including the effects of direct chemical oxidation as well as those of a further exothermic hydrolysis reaction and the evaporation and condensation of water. The model contains three main variables: the temperature of the body, the liquid water concentration in the body, and the water vapour concentration in the body. Chapter 4 investigates the limiting case of the model equations as the thermal conductivity and diffusivity of the body become large. In particular it is shown that, in this limiting case, the model can have at least twenty-five distinct bifurcation diagrams, compared with only two for the well known model without the effects of moisture content. In Chapter 5 the maximum principle and the methods of upper and lower solutions are used to derive existence, uniqueness and multiplicity results for the steady state solutions of the spatially distributed model. Finally, in Chapter 6, existence and uniqueness results for the time dependent spatially distributed model are derived.

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CONTENTS

CHAPTER 1	INTRODUCTION	1
1.1	Physical background	1
1.2	Formulation of the model for self-heating by a single exothermic reaction	3
1.3	Interpretation	6
CHAPTER 2	PRELIMINARY RESULTS AND A CASE STUDY	24
2.1	Preliminary results	24
2.2	Industrial case study	42
CHAPTER 3	THERMODYNAMIC DERIVATION OF A MODEL FOR THE SELF-HEATING OF DAMP CELLULOSIC MATERIALS	85
3.1	Heat producing reactions	85
3.2	Assumptions	86
3.3	Derivation of the equations	87
3.4	Dimensionless formulation of the equations	98
3.5	The steady state equations for the spatially distributed model	103

CHAPTER 4	THE SPATIALLY UNIFORM MODEL	106
4.1	Introduction	106
4.2	Questions of existence, uniqueness and multiplicity of solutions	107
4.3	The nature and stability of steady state solutions	117
4.4	Hopf bifurcations and periodic solutions	123
4.5	Plotting degeneracy and bifurcation curves: the pseudo-arclength method	133
4.6	The degeneracy curves in λ_1, L space	135
4.7	The distinct bifurcation diagrams	143
CHAPTER 5	EXISTENCE, UNIQUENESS AND MULTIPLICITY RESULTS FOR THE SPATIALLY DISTRIBUTED STEADY STATE MODEL	153
5.1	Existence results	153
5.2	Uniqueness results	170
5.3	Results on the multiplicity of solutions	184
CHAPTER 6	EXISTENCE AND UNIQUENESS RESULTS FOR THE TIME DEPENDENT PROBLEM	196
CONCLUDING COMMENTS		204
REFERENCES		205