The Jacobi triple product, quintuple product, Winquist and Macdonald identities

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Abstract

This thesis consists of seven chapters.
Chapter 1 is an introduction to the infinite products. Here we provide a proof for representing sine function as an infinite product. This chapter also describes the notation used throughout the thesis as well as the method used to prove the identities. Each of the other chapters may be read independently, however some chapters assume familiarity with the Jacobi triple product identity.
Chapter 2 is about the Jacobi triple product identity as well as several implications of this identity.
In Chapter 3 the quintuple product identity and some of its special cases are derived. Even though there are many known proofs of this identity since 1916 when it was first discovered, the proof presented in this chapter is new. Some beautiful formulas in number theory are derived at the end of this chapter.
The simplest two dimensional example of the Macdonald identity, A2, is investigated in full detail in Chapter 4. Ian Macdonald first outlined the proof for this identity in 1972 but omitted many of the details hence making his work hard to follow.
In Chapters 5 and 6 we somewhat deviate from the method which uses the two specializations to evaluate the constant term and prove Winquist's identity and Macdonald's identity for G2. Some of the work involved in proving G2 identity is new.
Finally in Chapter 7 we discuss the work presented with some concluding remarks as well as underlining the possibilities for the future research.
Throughout the thesis we point to the relevant papers in this area which might provide different strategies for proving above identities.

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